# The Effect of Large Jumps in a Ring-like Quantum Network

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*Abstract*—**Quantum communications promises major changes in today's communication networks by sending qubits over long distances. These qubits enable large-scale quantum computing or information-theoretically secure distribution of symmetrical keys. One of the main enablers is quantum teleportation, which makes sending quantum information between two nodes possible even when they are far apart. From these nodes, one can build a larger quantum network, but due to the nature of quantum physics, certain tasks that are well understood in classical networks, such as routing, cannot be handled in a similar way. Our work focuses on modifying a previously created model for a ring-like quantum network and assessing the effect of introducing a new node type. Our results show that this node can alter properties of the underlying network. We also look at the possibilities of modeling the capacity of the network as well as the availability of the newly introduced edges, which open interesting questions for future research.**

*Index Terms*—**quantum communication networks, entanglement, routing**

## I. INTRODUCTION

QUANTUM computing and communication use the laws of quantum physics to create a new field of technology, which in theory enables us to perform specific tasks better than what is achievable with our current classical systems. For example, in computing, quantum machines can efficiently break the RSA cryptosystem [1] by solving the factoring problem using Shor's algorithm [2]. Quantum random number generation [3], quantum sensing [4], and simulation [5] are also promising fields, which allow one to create good random numbers, to make more precise measurements, and to better understand certain physical systems [6]. For quantum communication, there are also promising new solutions, like quantum key distribution (QKD) [7], which makes it possible to share a secret key between two parties in such a way that they can even detect if an eavesdropper is present [8].

Another aspect of quantum communications is to connect nodes powered by quantum technologies, therefore creating a network that transmits quantum information. For example, the current implementations of quantum computers are in the NISQ era (Noisy Intermediate-Scale Quantum) and only offer a few hundred quantum bits (qubits), which are prone to suffering from different errors during computation and readout, significantly reducing their effectiveness. A possible way to tackle this problem (apart from creating more efficient and scalable error-correcting codes) is to connect these small-scale computers through a network, therefore creating a distributed but larger-scale machine [9]. A network like this is quite different from its classical counterpart and can enable new services for users while promising new challenges for researchers as well.

If one wants to send information encoded into a qubit (the information-carrying entity in quantum technologies) to another party, the most straightforward way is to send it through the appropriate medium. For example, if we encode the quantum information into the polarization of a single photon, then one can use an optical fiber for transmission. This can only work for a certain distance, because the so-called No-Cloning Theorem prohibits the perfect copying of an unknown qubit; therefore, we cannot use conventional optical amplifiers. To circumvent this problem, the communicating parties can utilize the quantum teleportation protocol [10]. This approach uses an entangled qubit pair and involves sending only two classical bits from one side to the other. This enables the receiving party to successfully recreate the unknown quantum state, but the original one will be destroyed (so the No-Cloning Theorem is not violated).

With this procedure, we can connect nodes, and they will be able to teleport qubits to their neighbors. If two non-neighboring nodes wish to exchange information, they will use the entanglement swapping protocol [11], where an entangled pair between the end nodes will be created with the help of the intermediate nodes, which can later be used for teleportation. From this, we create a quantum network consisting of nodes, which are connected by links capable of sending and receiving qubits (as well as classical information) and also performing certain operations on these qubits to implement teleportation and entanglement swapping.

As we have seen, quantum systems behave differently compared to their classical counterparts, which is also evident in quantum networks. One of the main problems in a communication network is routing, where one wants to find the best path under given circumstances from one node to another. In classical networks, this is a well-known problem with several different solutions. In the quantum case, we have to approach this task differently. Here, our main goal is to have an entangled pair of qubits between the source and the destination node. This means finding the right path along which entanglement swapping can be performed. However, the qubits used in the network must be handled carefully, so their state does not change during operations or transmission. Additionally, the time for which one can store them without losing their state is also limited. These requirements have to

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be taken into consideration, which means that the currently used classical algorithms need to be updated or new solutions have to be developed.

The physical implementation of a quantum network has been recently demonstrated, but with only a few nodes [12] [13]. Researchers also demonstrated the possibility of a quantum router [14]. The main challenges are to maintain the state of the qubits for extended periods of time, so they can be used during the protocols (this is the field of quantum memories) and to efficiently implement the required operations, which enables one to perform the necessary steps without introducing errors. Although the technology is not yet at a stage where large-scale quantum networks (like a quantum internet) can be created, it is nonetheless important to study them and to create algorithms for managing and controlling this new network type.

Our contribution is an extension of a previous model with a new type of node, which is motivated by real-world network topologies and improves specific characteristics of the underlying quantum network. We also carry out availability and capacity simulations to study the dynamic behavior of the network.

The structure of this paper is the following. Section II introduces the necessary background and the related work regarding routing in quantum networks. In Sec. III, we present the base model that we used in our work and also our extensions to it. Sec. IV details our results regarding the average route length and swap counts in Sec. IV-A, while Sec. IV-B introduces our capacity and availability modeling. At the end, Sec. V concludes our work.

## II. Overview of quantum internet protocols

To create a large quantum network, we need protocols, both quantum and classical, as well as quantum memories [15]. On the classical side: routing, signaling, or synchronization protocols [16] are necessary to guide and manage the quantum links. In contrast, the quantum protocols are, of course, essential for transmitting quantum information. Here, we introduce two quantum protocols and the related work regarding the routing problem.

*I. Quantum teleportation:* During teleportation [10], Alice would like to send a quantum state to Bob, without sending it over directly (note that this state can be unknown to Alice). For this operation, they will use a previously shared entangled pair of qubits, with one qubit being at Alice's side and the other at Bob's. Alice will first entangle the quantum state with their half of the pair and then measure them in a given basis. The measurement results will be sent to Bob on a classical channel (this means two bits), and based on these, Bob will perform certain operations on their qubit. At the end of the procedure, the qubit on Bob's side will be in the same state as the qubit that was meant to be teleported. As this state was measured by Alice, its superposition is lost and cannot be recovered. The act of entangling qubits and then measuring them (in a specific basis) is called the Bell measurement.

It is essential to note that we cannot create a faster-than-light communications protocol using teleportation, as the information necessary for Bob to perform the given operations must be sent over a classical channel. Without it, Bob will only have the right state with probability less than one.

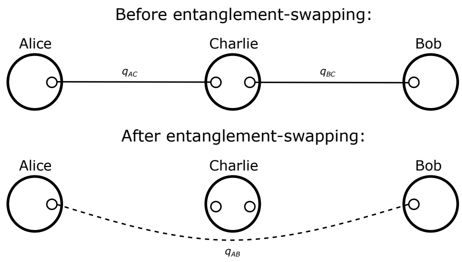



Fig. 1. Entanglement swapping with three participants. Alice and Charlie, as well as Charlie and Bob, have previously created entangled pairs denoted by $q_{AC}$ and $q_{BC}$. After Charlie performs the Bell measurement, Alice and Bob will have an entangled pair $q_{AB}$ between them, and they will be directly connected to each other.

*II. Entanglement swapping:* In the context of entanglement swapping [11], we have three parties: Alice, Bob and Charlie. Alice and Bob are not directly connected, but both of them have Charlie as their neighbor. A connection here means that they share a quantum and a classical channel, on which they can perform the previously introduced quantum teleportation protocol. The consequence of this requirement is that they have a shared entangled pair of qubits for every connection. The starting position of the protocol can be seen at the top of Fig. 1.

Charlie will perform the Bell measurement on their qubits, belonging to separate entangled pairs. After the measurements, the results will be sent to either Alice or Bob, allowing them to make the necessary corrections. After these steps are done, Alice and Bob will have a shared entangled pair. This means that they will be connected, even though they were not neighbors beforehand. In this setup, Charlie can be thought of as a quantum relay node. The final state of the three nodes is depicted at the bottom of Fig. 1. Using this technique, if there is a line of nodes in a quantum network, the nodes at the ends of this path can be easily connected by completing the swap protocol on the intermediate nodes.

One of the first works about routing entanglement through a quantum network was presented in [17], where the authors extended Dijkstra's algorithm to maximize the number of entangled pairs between two endpoints. After this, most work focused on the same problem in the context of a specific network structure. In [18] the researchers used ring and sphere-like networks (this work is based on this model) to create a framework for entanglement routing, the authors of [19] studied a diamond-shape structure, while [20] and [21] looked at specific lattice and grid-based topologies, where routing decisions can be easier. Quite recently, there were a number of publications regarding routing in an arbitrary graph while also supporting multi-path routing (in this problem, we have to serve multiple source-destination pairs) [22]: [23] uses fidelity as its base metric, while [24] introduces the "cost-vector analysis".

Another approach is the stochastic modeling of the quantum network, which can incorporate the different imperfections of the implementation as well as the probabilistic nature of quantum physics. In [25], the authors studied a line of repeaters, while in [26] they focused on a star-like topology.

It is also beneficial to look at specific working environments, where entanglement routing might be used, as it might provide more possibilities for routing. For example, routing in a satellite-based network [27] or in the case of faulty quantum memories [28].

As the physical implementation and the theoretical study of quantum networks progress, it is also important to create a common framework or network stack, which can be used to manage larger networks and upon which applications can be built [29] [30].

## III. The base model and our extensions

### A. The base model

The base model that we modified during our work was created by Schoute et al. [18], and the authors' main goal was to create a simple network structure that makes the routing decision easier. They created a ring and sphere-like network, but we focus only on the ring-based approach.

The graph $G_n = (V_n, E_n),\ n \in \mathbb{N}$ is a network with $N = 2^n = |V_n|$ nodes and undirected edges. The nodes in the graph are labeled mod $2^n$, that is $V_n = \{0, 1, \ldots, N-1\}$ and an edge $e = (\alpha, \beta)$ with $\alpha, \beta \in V_n$ is part of $E_n$ if:

$$|\alpha - \beta| \equiv \mathrm{gcd}_2(\alpha, \beta) \mod 2^n,$$

where $\mathrm{gcd}_2(\alpha, \beta)$ is the largest power of two that divides both $\alpha$ and $\beta$.

From this it can be seen that the $N$-long circle $C_N$ is a subgraph of $G_n$ and if $\alpha$ is divisible by $2^k$, then there is and edge to $\alpha \pm 2^k \pmod N$, which makes it possible to skip $2^k$ nodes on $C_N$. The authors of [18] also grouped the edges into two categories. The edges of $C_N$ correspond to physical links in the network, that is, a connection where quantum teleportation can be performed. Any other edge not present in $C_N$ is a virtual quantum link (or VQL), which can be created from the physical links with the help of entanglement swapping. On Fig. 2 we can see the graph $G_2$ and $G_3$. The solid edges represent physical links, while the dashed ones are VQLs created from these links.

It can also be proven (and it is evident from the example of $G_2$ and $G_3$) that $G_{n-1}$ is also a subgraph of $G_n$. Furthermore there is an elegant recursion step which can be used to generate $G_n$ from $G_{n-1}$: For every physical link $e_{\alpha,\beta}$ $,\forall \alpha, \beta \in V_{n-1}$ and $|\alpha - \beta| = 1$ in $G_{n-1}$ create a new node $\gamma$ and two more edges $(\alpha, \gamma)$ and $(\gamma, \beta)$. After relabeling the nodes from 0, these new edges will be the physical links in $G_n$, and the "old" physical links will become new VQLs.

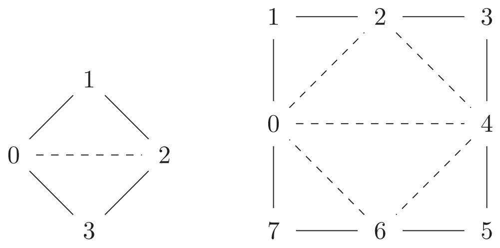
In [18] a number of different properties for $G_n$ was proven, but the most important is about the diameter of the graph (the longest path from all possible shortest paths), which is only $d(G_n) = O(\log N) = O(n)$ compared to $d(C_N) = O(N)$.

### B. Extensions to the base model

We extended the base model in the following way. Although the diameter of the graph significantly changed compared to $C_N$, there are still nodes, the ones with an odd label, which are not part of a smaller circle. To overcome this property, we extend the base model with a new node, called the central node $C$. The central node can connect to any of the nodes in $G_n$ through a new physical link, and with the help of entanglement swapping, it can also create new VQLs. Our primary goal with this node type is to further reduce the diameter (similar to the small-world property) and also the number of swap operations required for a given source-destination pair. If we think of the nodes in $G_n$ as base stations in a telecommunications network, the original physical links and VQLs can be viewed as common interfaces, while the new central node represents a possible connection to the backbone network.

A central node can have at most $k \leq N$ edges, and as the subgraph of physical links belonging to $C$ has a star topology, the new VQLs generated by entanglement swapping create a complete graph with $k$ nodes. An example of $G_4$ with a central node $C$ is presented in Fig. 3. The original VQLs of $G_4$ are not shown for better readability.

It is also important to discuss the extensions needed during the routing process. In [18], the main part of the routing algorithm is the `path`$(\alpha, \beta)$ function, which gives back the shortest path between nodes $\alpha$ and $\beta$. This function calls the `path2`$(\alpha, \beta)$ and the `bestMove`$(\alpha, \beta)$ subroutines. The `path2` function gives back the shortest path of length at most 2, if there is any, while the `bestMove` gives back the next




(a) The graph $G_2$ (b) The graph $G_3$

Fig. 2. The graphs $G_2$ and $G_3$ created by the definitions given in [18]. The solid edges represent physical links, while the dashed edges are VQLs (Virtual Quantum Links) created by entanglement swapping along the physical ones.

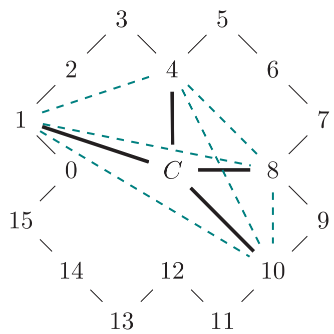
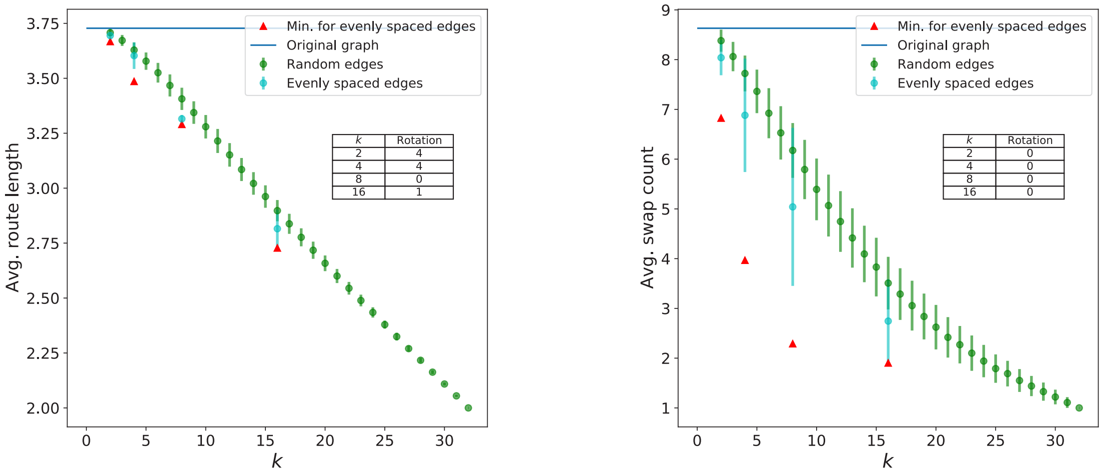



Fig. 3. A possible central node ($C$) with four new physical edges (solid lines) and all possible VQLs (teal dashed lines) in $G_4$. Note that the original VQLs of $G_4$ are omitted.




(a) (b)

Fig. 4. The average route length (a) and swap count (b) with their standard deviation for $G_5$ with two central node types. The horizontal line shows the values for the original $G_5$. For the case of a central node with evenly spaced edges, the optimal rotation value is also shown on the figures and summarized in a table. Sample size: 1000

node, which is guaranteed to be on the shortest path between $\alpha$ and $\beta$.

With the introduction of the central node, it is also necessary to extend these two subroutines. The `path2` will now check if the nodes are neighbors through $C$, while the `bestMove` will also examine the possibility of whether we can get closer to $\beta$ if we go through the central node (this is only considered if $\alpha$ is connected to $C$).

## IV. Results and Discussion

### A. The effect on average route length and swap counts

To measure the effect of a central node on $G_n$, we first looked at two important properties of the newly created quantum networks. The first one is the average route length (or average path length), which is different from the diameter of the graph. This is an important characteristic of any network, as it measures how easily one can navigate the network and reach other nodes. A lower average route length indicates that most nodes are accessible from any point in the network in fewer steps, which helps the flow of information. The second one is the average swap count required to create an entangled pair between a given source-destination pair. It is desirable to keep the average swap count as low as possible, as the entanglement swapping protocol is a complex task and needs additional classical communication to succeed, which adds delay to the overall communication step. Calculating the swap count on the original $G_n$ for a given route can be done in the following way: If the path is $v_1, v_2, v_3, \dots, v_k$ with $v_i \in V_n$ $\forall i = 1, \dots, k$, then the swap count that is necessary to "get" from $v_1$ to $v_2$ is distance of the two nodes on the ring $C_N$ minus 1.

If we extend the graph with a central node, then we have to check for every consecutive node in the path whether they are connected through the central node or not. If they are, the swap count needed is just 1. After this, the swap count for the entire path can be calculated by adding up the swaps for the subsequent steps and adding one, as a final swap is necessary at the end to create the desired connection.

During our simulation, we looked at two types of central nodes. The first one is a node with $k \leq N$ random edges, which connect to different nodes on $G_n$. The second one is a central node that has $k \leq N/2$ evenly spaced edges connecting to the nodes. This means that the central node is connected to nodes $0, \frac{N}{k}, 2 \cdot \frac{N}{k}, \dots, (k-1) \cdot \frac{N}{k} \pmod N$ with $k$ in the form $2^i \quad i = 1, \dots, n-1$ or one of its possible circular rotations (e.g. starting from a different node).

For the random central node, we sampled possible edges for a given $k$. For the evenly spaced case, we examined possible values of $k$ and their corresponding circular rotations. After this, we calculated the route length and the swap count for every possible source-destination pair in $G_n$ and averaged the results, which can be seen in Fig. 4. The results show that the introduction of the central node has reduced both the average route length and the average number of swaps. For large $k$ (close to $N$), this is not surprising, because the addition of a node with this many edges creates an almost complete or a complete graph of VQLs on the nodes in the network. It is also important to notice that the standard deviations of the average route length and swap count are relatively small for central nodes with random edges. This is not the case for evenly spaced edges, as the different rotation values have a strong influence on the achievable average swap counts.

We can observe that in the lower interval of $k$, the best results for both values occur when the central node has $N/2$ evenly spaced edges, meaning that every second node is connected to $C$. In this case, the rotations (there are two possibilities) do not significantly alter the average route length, and we can calculate the diameter of the network, as stated in Proposition 1, along with a sketch of the proof. It is also important that for the evenly spaced edges, the average swap count is minimal if we only connect even nodes. The reason for this comes from the structure of $G_n$. Even nodes are connected to each other through VQLs and have high degree counts; therefore, they are important during path selection. But the VQLs connecting them can only be realized by a high number of swap operations. For example, realizing the dashed line between node 0 and 4 in $G_3$ (depicted in Fig. 2) requires three swap operations. With the help of a central node that has evenly spaced edges, we can create new VQLs between the even nodes, which require only one swap, thereby reducing the average swap count needed during routing.

**Proposition 1.** *The diameter of $G_n$ extended with a central node $C$ that connects all odd or even nodes is 3.*

*Proof.* In $G_n$, there are three possible source-destination types: even-to-even, odd-to-odd, and even-to-odd (odd-to-even is the same because of the symmetry of this particular network). For odd-to-odd, if $C$ connects all odd nodes, then the route length is 1. For even-to-odd nodes, we can proceed from the source to one of the neighboring odd nodes and then jump to the destination, resulting in a route length of 2 or 1 if they are originally directly connected. This is also true if $C$ connects all even nodes, as in that case we can always choose an even node neighboring the destination. For the even-to-even routes, there are three possibilities:

1) If the source and destination are connected by a VQL, then the route length is 1.
2) If their distance was 2 on $G_n$, then the route length remains the same.
3) If their distance was $\geq 3$, then we can once again go to one of the odd neighbors, then jump to the odd node nearest to the destination (there are just two possibilities) and do one more step to the destination itself. This is a path of length 3.

The same can be said about odd-to-odd pairs if $C$ connects the even nodes.

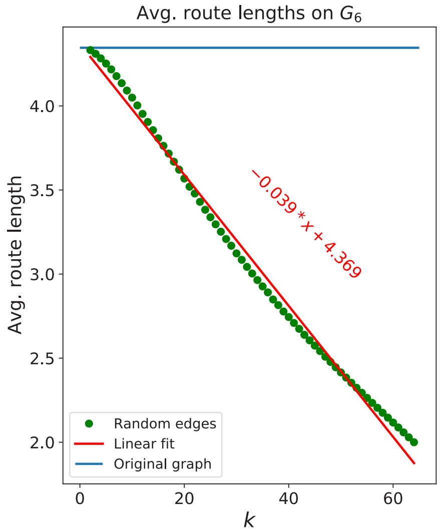
□

Another important finding is regarding the shape of the two graphs corresponding to the random edge case. This is more evident on larger networks. The average route length exhibits a linear trend, while the average swap count follows a decaying exponential pattern. This is shown in Fig. 5, where the values are plotted for $G_6$. The average route length was fitted with a linear function of the form $ax + b$, and for the average swap count, we used the form $ae^{-bx} + c$. Both of them provide a good fit to the data points (the exact values can be seen in the description of Fig. 5). The exact reason for the shape of these graphs is a promising future question.

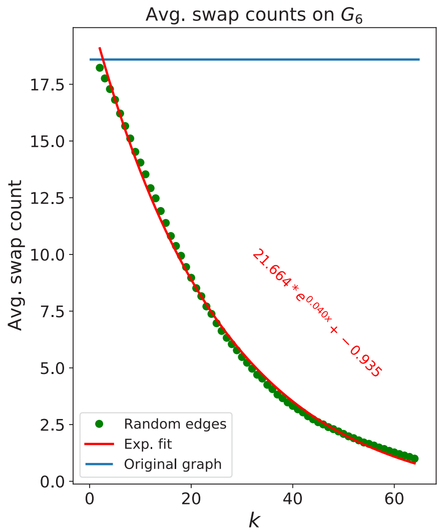
### *B. Capacity and availability modeling*

As we saw in the previous section, the central node with a large degree has significant effects on the properties of the underlying network. This means that the central node has to have a large enough quantum memory to hold at most $N$ qubits and a long enough decoherence time (the time after which the qubits lose their state) for using them as VQLs. However, the current physical implementation of quantum memories can only hold a few qubits at a time [15]. This means that the more edges the central node has, the less usable the VQLs become.

To model this phenomenon, we propose two simple methods. The first one gives a capacity $E$ to the central node. This means that it can serve $\leq E$ nodes reliably. If $C$ has $k$ edges, the current usage is given by $E/k$. To show the

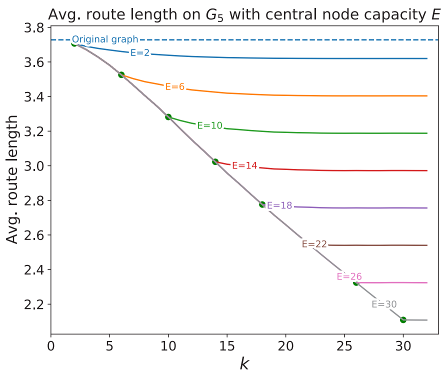



(a)




(b)

Fig. 5. Curve fit for the average route length (a) and swap count (b) for $G_6$. For the average route length, we used a linear function, and for the swap count, an inverted exponential function. In the case of the average route, the standard deviation errors for the parameters are 0.0004 and 0.0163, while for the average swap count, they are: 0.1351, 0.0008, and 0.1567. Sample size: 1000

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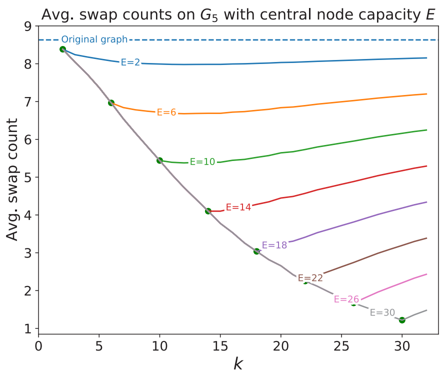
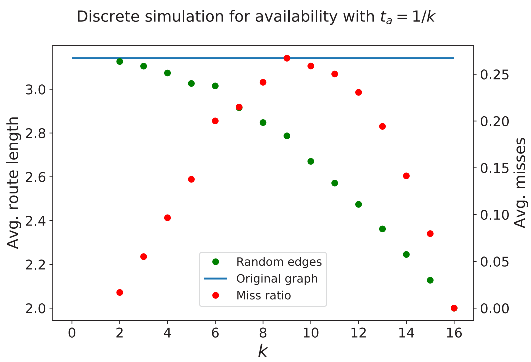
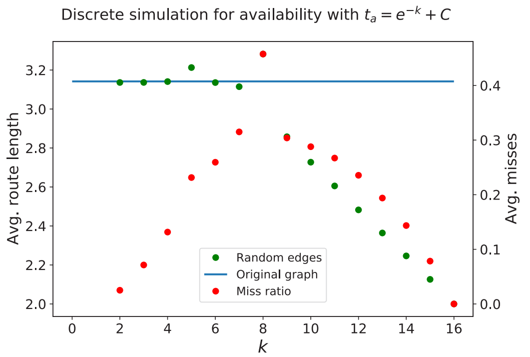



(b)

Fig. 6. Capacity modeling of the central node $C$ with different capacity values $E = 2, 6, \dots, N-2$. In (a), we can see the average route length, while (b) shows the average swap count. Sample size: 5000.




(a)




(b)

Fig. 7. Discrete time simulation results for the availability of the VQLs on $G_4$. On (a) the function of the availability time is $t_\mathrm{a} = 1/k$, while on (b) it is given by $t_\mathrm{a} = e^{-k} + c$, where $c$ is a constant to limit the value of $t_\mathrm{a}$ as $k \to N$. In both (a) and (b) $t_\mathrm{r} = 1$. Sample size: 5000

unreliability of larger quantum memories, we take the linear combination of the average route length of the original graph $G_n$ and the average route length of $G_n$ extended by a central node with $k$ random edges using weights $1 - e_k$ and $e_k$, where $e_k = \min(1, E/k)$. This combination ensures that a central node with large capacity can serve all its neighbors, but if the capacity is lower (e.g., the quantum memory used is not reliable), the properties of the network get closer to the original graph as $k$ increases. The effect of this method with different $E$ capacities can be seen in Fig. 6.

Regarding the average route length, the graph follows the previous results until $k \leq E$; after this, it does not improve significantly, but we can still observe a slight drop as $k$ gets larger. This is caused by the previously described large gains introduced by a central node with random edges. The average swap count behaves differently. Here, after $k$ gets larger than $E$, the graphs take a steep incline towards the original value for $G_n$. This effect is more noticeable when $k$ is closer to $N$, because in this interval the average swap counts are in a slower decline.

For the availability modeling, we created a discrete simulation. The central node with $k$ edges can serve requests for $t_\mathrm{a}$ time, after this, it takes $t_\mathrm{r}$ time to make the VQLs available again. The value of $t_\mathrm{a}$ and $t_\mathrm{r}$ can depend on the edges connected to the central node. We calculate the route step by step, which takes 1 unit of time. If the current node in the path calculated that the next hop is through the central node, it tries to access the VQLs. If it succeeds, the routing takes this VQL; if the request fails, we take the link that is given by the next hop on the original graph. We gather the average route length as well as the ratio of missed resource requests. The results of the simulation can be seen in Fig. 7 with different functions for $t_\mathrm{a}$.

The most significant outcome in the graphs is that as the degree of the central node increases, the number of unsuccessful requests also grows. This has an effect on the average route length, as we have to use the VQLs that are present on the original graphs in a larger portion of the time. At larger $k$ values, this negative effect quickly fades away. The reason for this is that at larger degree values, there is a higher probability that the source-destination pair is connected directly or that they are only a few hops away. Since the simulation starts at zero, the first request is successful.

## V. Conclusion

In our work, we focused on entanglement generation in a quantum network. We introduced an extension to an existing model that adds a central node and examined the impact it had on various properties of the underlying network. Our results show that these effects are significant, reducing both the average route length and the swap count required to connect a source-destination pair. As the current physical implementations of quantum memories are not yet at a stage where they can be reliably used in a quantum network, we have created two simple models to incorporate the capacity of a node or the availability of a quantum link. Based on these models, our simulations show that even a central node with a few edges can improve the network.

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