

# Establishing Lower Bounds on the Peak-to-Average-Power Ratio in Filter Bank Multicarrier Systems

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**Abstract**—Filter bank multicarrier (FBMC) modulation is a promising candidate as the primary physical layer waveform in next generation broadband networks. However, FBMC, like other multicarrier schemes, suffer from high peak-to-average power ratio (PAPR). Due to inherent overlapping of the time domain FBMC symbols, the methods designed for PAPR reduction in orthogonal frequency division multiplexing (OFDM) can not be applied in a straightforward manner for FBMC. In this paper, an optimization method is proposed to obtain significant PAPR reduction for short FBMC frames. The formulation of the optimization problem enables the use of both tone reservation and active constellation extension, which are well-known methods for OFDM PAPR reduction. The results establish a practical lower bound on achievable PAPR with these two approaches.

**Keywords**—filter bank multicarrier, peak-to-average power ratio, optimization, constellation extension, tone reservation

## I. INTRODUCTION

Multicarrier modulation schemes gained considerable attention during the past decades in broadband mobile and fixed wireless systems. Numerous standards adopt orthogonal frequency division multiplexing (OFDM) as their physical layer modulation scheme. Among others, various IEEE 802.11 standards, 3GPP LTE, or in the broadcast world, DVB-T2. Evolving present systems in terms of spectral efficiency and advanced spectrum access techniques motivates the search for advanced multicarrier modulation techniques [1]. A family of a promising prospective waveforms is filterbank multicarrier (FBMC), from which we consider the specific scheme known as OFDM/OQAM.

FBMC shares many advantages with traditional OFDM, e.g., high spectrum efficiency, with simple access to both time and frequency domains in terms of scheduling etc. Specifically it offers very low adjacent channel leakage levels. However, FBMC signalling, like other multicarrier schemes, suffer from dynamic fluctuations in the instantaneous power, which is often described in terms of the peak-to-average-power ratio (PAPR). To mitigate this problem, PAPR reduction methods are being employed. Some of these techniques are directly inspired by OFDM based approaches. Such schemes include selective mapping [2] and clipping based [3] [4] methods. However, as adjacent FBMC symbols overlap in time by design, direct

adoption of PAPR reduction schemes originally conceived for OFDM generally yields suboptimal results. Recent algorithms specifically developed for FBMC are taking the overlapping nature into account, e.g., they employ multi block joint optimization [5], or alternative signal method [6].

In this paper we investigate two PAPR reduction schemes and their joint use for FBMC, namely tone reservation (TR) and active constellation extension (ACE). The basic concept of these methods as well as the formulation of the optimal solution through a quadratic constrained quadratic program (QCQP) are introduced for OFDM systems in [7] and in [8], respectively. A suboptimal program is solved for the joint use of the schemes in [9], whereas the program leading to optimal solution for ACE is shown in [10]. A clipping based implementation of TR and ACE was shown in [4], and ACE smart grid projection of Krongold has been also extended to FBMC in [11]. A sliding window based TR technique for FBMC is also shown in [12].

Although straightforward application of TR and ACE for FBMC apparently show some reduction in PAPR, they remain heuristic approaches and lack the basis of objective comparison. The gains in PAPR reduction with these techniques are significantly lower when compared to what can be achieved for OFDM with the same parameters i.e. same number of subcarriers, modulation alphabet, clipping ratio etc. Since FBMC symbols overlap in time, the problem cannot be solved on a per symbol basis (like in OFDM systems). Thus, an entire frame has to be treated.

Motivated by this apparent performance gap, in this paper we formulate the PAPR minimization problem for FBMC, taking into account the inherent intersymbol interference in the scheme. We apply either TR or ACE or both to achieve the smallest possible PAPR for an entire frame. We show that the problem can be cast as a QCQP and the optimum solution can be found numerically. This result provides the minimum achievable PAPR using TR and/or ACE, thus serving as a lower bound to the PAPR, to which practical, real-time implementation of TR and ACE can be compared to.

In real-world systems (e.g., in 3GPP LTE), the frames consist of a relatively small number of symbols to enable a fine-grained resource allocation. Therefore we can choose realistic frame sizes consisting of 8 and 12 symbols per frame and solve the optimization problem for these values.

Besides serving as a baseline for the amount of achievable PAPR reduction, a further potential use of the presented method is to construct optimized packet headers offline with

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minimal PAPR in order to transmit the headers with increased average power (e.g., for timing and frequency offset correction or channel estimation).

The paper is organized as follows. In section II the OFDM and FBMC signal synthesis and the PAPR metric is defined. In section III a the principles of PAPR reduction with TR and ACE schemes are summarized. In section IV the PAPR problem formulation for OFDM and the proposed formulation for FBMC signals are detailed. In section V the results of problem solution is shown and the practical lower bound of the PAPR reduction schemes are presented.

## II. SIGNAL MODEL AND PERFORMANCE METRICS

### A. Orthogonal frequency division multiplexing signal

The synthesis of the time domain signal is based on inverse discrete Fourier transform (IDFT) of data symbols chosen from a M-QAM modulation alphabet. Generation of a transmitted OFDM symbol is made the following way:

$$x[k] = \sum_{n=0}^{N-1} s_n e^{j \frac{2\pi}{NV} nk}, \quad (1)$$

where  $j = \sqrt{-1}$ ,  $N$  is the number of subcarriers and  $s_n$  is the complex data symbol on the  $n^{\text{th}}$  subcarrier and  $k$  is the discrete time and  $V$  is the oversampling rate.

### B. Filter bank multicarrier signal

A thorough introduction and comparison with OFDM can be found in [13]. Based on [13], a frame of the transmitted FBMC signal can be written as:

$$x[k] = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} (\theta^n s_{re,n}[m] p_0[k - mNV] + \theta^{n+1} s_{im,n}[m] p_0[k - mNV - \frac{NV}{2}]) e^{j \frac{2\pi}{NV} n(k - mNV)}, \quad (2)$$

where  $\theta^k = e^{j \frac{\pi}{2} k}$  and  $s_{re}[m]$  and  $s_{im}[m]$  denote the real and imaginary parts of the PSK or QAM modulating symbols on the  $n^{\text{th}}$  subcarrier in the  $m^{\text{th}}$  signalling time interval, respectively. The prototype filters  $p_0[m]$  are designed with an impulse response length  $L = KNV$ , where  $K$  is the overlapping factor, determining the span of the deliberate intersymbol interference in FBMC symbols. An FBMC frame is assumed to consist of  $M$  symbols (thus, the length of a frame is  $T = L + (M - 1)NV + \frac{NV}{2}$ ). Throughout this paper the choice of the prototype filter is identical to the one used in [14], known as the PHYDYAS filter, with  $K = 4$ . The synthesis of the signal written in (2) can be implemented efficiently using the inverse fast Fourier-transform (IFFT) and polyphase decomposition of the modulated prototype filters, as presented in [15]. A more intuitive representation of the synthesis is depicted in Fig. 1. The main blocks of the signal construction is the IFFT and subsequent filtering which is done separately for the real and imaginary parts of the frequency domain symbols whereas the transformed version of the imaginary part is added with a staggering (i.e., time delay) of  $\frac{NV}{2}$  relative to the real part of the signal.

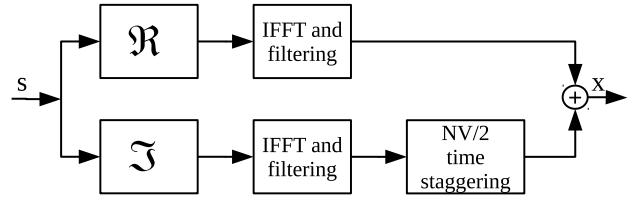


Fig. 1. Synthesis of FBMC signal using IFFT, filtering and time staggering

### C. Peak-to-average-power ratio

PAPR is a common metric to characterize the amplitude fluctuations of the signal. Highly dynamic fluctuations increase the required power reserve (back-off) of power amplifiers. Distortions caused by the nonlinear behaviour of the amplifier result in increased out of band radiation (spectrum regrowth), which defeats one of the primary goals of FBMC (i.e., very low adjacent channel leakage) as well as causing intercarrier interference which further degrades the quality of transmission.

The PAPR (in decibel) for OFDM is defined on a per symbol basis as follows:

$$\text{PAPR}_1 = 10 \log_{10} \left( \frac{\max_{0 \leq k \leq NV-1} \{|x[k]|^2\}}{\frac{1}{NV} \sum_{k=0}^{NV-1} |x[k]|^2} \right), \quad (3)$$

where  $|\cdot|$  is the magnitude of the signal.

On the other hand, the overlap between adjacent symbols in FBMC raises a question about unambiguously defining the PAPR of FBMC. This issue is further complicated by the presence of filter transients at the beginning and end of each frame.

For FBMC only the steady state part is taken into account, thus the first and last  $L$  samples (which are transient parts caused by the filters) are discarded. This means that it only makes sense to calculate the PAPR for at least  $2K + NV$  samples. After discarding the transients, the rest of the signal is divided into blocks of length  $NV$ . This provides a good comparison with OFDM-like signals where the symbols are orthogonal to each other. The PAPR for FBMC is then defined as:

$$\text{PAPR}_2 = 10 \log_{10} \left( \frac{\max_{mNV \leq k \leq (m+1)NV-1} \{|x[k]|^2\}}{\frac{1}{NV} \sum_{k=mNV}^{(m+1)NV-1} |x[k]|^2} \right), \quad (4)$$

$$K \leq m \leq M - K - 1.$$

## III. PAPR REDUCTION

An approach to reduce the PAPR is to minimize the numerator of (3) and (4) for OFDM and FBMC, respectively. To address this problem, the "correction term"  $y$  is added the original signal  $x$  and then  $E$ , the instantenous energy of the

complex baseband signal is minimized. The general problem formulation becomes

$$\begin{aligned} & \text{minimize } E \\ & |x[k] + y[k]|^2 \leq E, \quad 0 \leq k \leq T - 1. \end{aligned} \quad (5)$$

Both the TR and ACE schemes fit into this framework by posing constraints on  $y$ . The demodulated (frequency domain) symbols corresponding to  $y$  are denoted with  $t$ . Then TR and ACE methods can be formulated in the following way:

#### A. Active constellation extension

The general criterion for ACE restricts the Euclidean distance between the constellation points being not smaller than the original distance. For QPSK modulation the original points and the corresponding extension regions are depicted in Fig. 2 with black dots and grey shading, respectively. For 16QAM alphabet the rule is more sophisticated. For 16QAM the extension regions are shown in Fig. 3. First of all the four closest constellation points to the origin are not involved (since the movement in any direction would decrease the distance to some of the other points), these are marked with white circles in the figure. For the four corner points the same rule applies as for QPSK. The rest of the points have one degree of freedom, thus they can move outwards in the directions depicted with dashed lines. If the minimization in (5) is performed applying the ACE constraints, the original symbols  $s$  are modified to  $t^{ACE} = s + t$  in order to obtain the PAPR reduced signal.

#### B. Tone reservation

In the tone reservation scheme a number of tones/subcarriers, are reserved specifically for PAPR reduction purposes and thus these carriers do not take part in data transmission. Let  $t_R$  be the set of reserved subcarrier indices and  $n_{res} = |t_R|$  the number of reserved tones. The symbols on the reserved carriers are zero initially  $s_n = 0$  for  $n \in t_R$ . Then the criterion for TR becomes

$$t_n^{TR} = \begin{cases} s_n, & \text{if } n \notin t_R \\ \text{arbitrary}, & \text{if } n \in t_R \end{cases}. \quad (6)$$

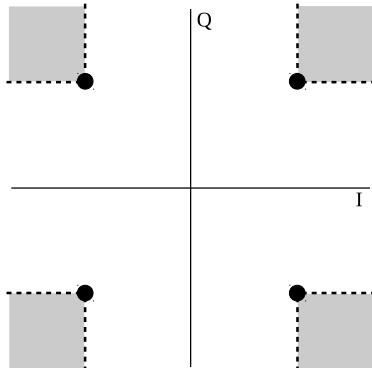


Fig. 2. ACE extension regions for QPSK modulation

#### C. Joint use of TR and ACE

The constraints for TR are more permissive on the reserved tones than those imposed by the ACE, thus the two methods can be used jointly, ideally resulting in even lower PAPR values. In this case the constraints are combined in the following way:

$$t_n^{TRACE} = \begin{cases} t_n^{ACE}, & \text{if } n \notin t_R \\ t_n^{TR}, & \text{if } n \in t_R \end{cases}. \quad (7)$$

Both schemes share the same favourable property that the receiver architecture does not need any modifications, only the indices of the reserved tones must be known in the receiver in order to discard them. However, both schemes increase the average power of the transmitted signal and TR introduces data rate loss since the reserved tones do not carry useful data.

## IV. PROBLEM FORMULATION AND OPTIMAL SOLUTION

#### A. Problem formulation for OFDM

Let  $F^{-1}$  be the  $NV \times N$  IDFT matrix. Then we can reformulate (1) using matrix operations to  $x = F^{-1}s$ . Using this notation the QCQP for PAPR reduction to a single OFDM symbol can be formulated based on [7] as:

$$\begin{aligned} & \text{minimize } E \\ & \text{subject to} \\ & [x \quad F^{-1} \quad -I] \begin{bmatrix} 1 \\ t \\ a \end{bmatrix} = 0 \\ & E \geq |a_k|^2, \end{aligned} \quad (8)$$

where  $a$  is an auxiliary variable to express the amplitude of the signal. The instantaneous magnitude  $|a_k|^2$  is upper bounded by  $E$  which is minimized. The described for ACE in III-A and TR in (6) are applied on  $t$ :

- TR:  
 $-\infty \leq t_{t_R} \leq \infty, t_R = \{\text{reserved indices}\}$

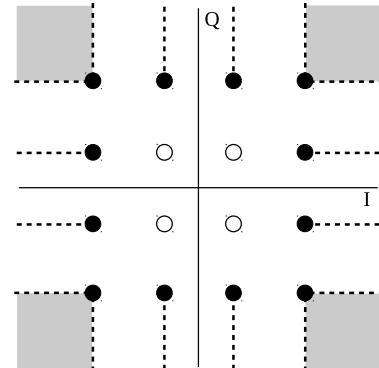


Fig. 3. ACE extension regions for 16QAM modulation

- ACE (QPSK):
 $\Re\{t_{R_P}\} \geq 0, R_P = \{n \mid \text{sign}(t_{re,n}) = 1\},$ 
 $\Re\{t_{R_N}\} \leq 0, R_N = \{n \mid \text{sign}(t_{re,n}) = -1\},$ 
 $\Im\{t_{I_P}\} \geq 0, I_P = \{n \mid \text{sign}(t_{im,n}) = 1\},$ 
 $\Im\{t_{I_N}\} \leq 0, I_N = \{n \mid \text{sign}(t_{im,n}) = -1\}.$

- ACE (16QAM):

corner points:

$$\begin{aligned} \Re\{t_{R_P}\} &\geq 0, & R_P &= \{n \mid t_{re,n} = 3\}, \\ \Re\{t_{R_N}\} &\leq 0, & R_N &= \{n \mid t_{re,n} = -3\}, \\ \Im\{t_{I_P}\} &\geq 0, & I_P &= \{n \mid t_{im,n} = 3\}, \\ \Im\{t_{I_N}\} &\leq 0, & I_N &= \{n \mid t_{im,n} = -3\}. \end{aligned}$$

additionally for side points:

$$\begin{aligned} \Im\{t_{R_{S1}}\} &= 0, & R_{S1} &= \{n \mid t_{re,n} = 1\}, \\ \Im\{t_{R_{S2}}\} &= 0, & R_{S2} &= \{n \mid t_{re,n} = -1\}, \\ \Re\{t_{I_{S1}}\} &= 0, & I_{S1} &= \{n \mid t_{im,n} = 1\}, \\ \Re\{t_{I_{S2}}\} &= 0, & I_{S2} &= \{n \mid t_{im,n} = -1\}. \end{aligned}$$

inner points:

$$t_Z = 0, \quad Z = \{n \mid t_{re,n} = 1 \wedge t_{im,n} = 1\}.$$

Since the two constraints – ACE and TR – are independent, they can be applied at the same time as well.

### B. Problem formulation for FBMC

In order to extend the problem formulation to FBMC signals we propose a way to express FBMC signal synthesis in a linear algebraic form. First, polyphase decomposition allows us to express the filtering procedure with a single matrix. We denote the filter matrix as

$$P = \begin{bmatrix} \text{diag}(p[0], p[1], \dots, p[NV-1]) \\ \text{diag}(p[NV], p[NV+1], \dots, p[2NV-1]) \\ \vdots \\ \text{diag}(p[(K-1)NV], p[(K-1)NV+1], \dots, p[KNV-1]) \end{bmatrix},$$

where  $p[i]$  is the  $i^{th}$  tap of the prototype filter and  $\text{diag}(\cdot)$  denotes the diagonal matrix of the elements. There are  $K$  pieces of  $NV \times NV$  submatrix resulting in  $P$ , a  $KNV \times NV$  matrix. As seen in the FBMC signal definition in (2) the symbols are also phase rotated with  $\theta$ , thus the  $P$  filter matrix is multiplied by such a phase rotation matrix accordingly:

$$\begin{aligned} H_1 &= P \cdot F^{-1} \cdot \text{diag}(\theta^0, \theta^1, \dots, \theta^{N-1}) \text{ and} \\ H_2 &= P \cdot F^{-1} \cdot \text{diag}(\theta^1, \theta^2, \dots, \theta^N), \end{aligned}$$

thus we obtain matrices to construct the two staggered portions, respectively. To embed the staggering to the matrix operation we append zeros to the matrices:

$$H'_1 = \begin{bmatrix} H_1 \\ 0_{NV/2 \times N} \end{bmatrix}, \quad H'_2 = \begin{bmatrix} 0_{NV/2 \times N} \\ H_2 \end{bmatrix}.$$

The zeros at the beginning of  $H'_2$  introduce the time shift, whereas the zeros at the end of  $H'_1$  are needed to match the dimensions with  $H'_2$ , so the inphase and the staggered part can be summed to create an FBMC signal. In order to express the final synthesis operator – whereby an arbitrary number of FBMC subsymbols can be summed up – we define the auxiliary matrices

$$H_A^q = \begin{bmatrix} 0_{NV \times qN} \\ H'_1 \\ 0_{NV \times (M-q-1)N} \end{bmatrix}, \quad H_B^q = \begin{bmatrix} 0_{NV \times qN} \\ H'_2 \\ 0_{NV \times (M-q-1)N} \end{bmatrix}.$$

With these matrices an FBMC subsymbol can be synthesized with a 'tail' of zeros of  $M-1$  subsymbol length. The  $M$ -fold augmentation leads to the band matrices

$$\begin{aligned} H_{F1} &= \left[ \begin{array}{c|c|c|c} H_A^0 & H_A^1 & \dots & H_A^{M-1} \end{array} \right], \\ H_{F2} &= \left[ \begin{array}{c|c|c|c} H_B^0 & H_B^1 & \dots & H_B^{M-1} \end{array} \right]. \end{aligned}$$

Thus the synthesis of an entire frame consisting of  $M$  subsymbols can be expressed as

$$x = H_{F1}s_{re} + H_{F2}s_{im}, \quad (9)$$

where  $s_{re}$  and  $s_{im}$  are the real and imaginary parts of the frequency domain symbols  $s$  for the whole frame, respectively, i.e., for  $M$  subsequent symbols. For a better understanding, the first term of the addition in (9) is illustrated in Fig. 4. The second term of the addition is similar except  $H'_1$  and  $s_{re}$  are replaced with  $H'_2$  and  $s_{im}$ , respectively.

Let  $t = t_{re} + j \cdot t_{im}$  represent the frequency domain symbols throughout an entire frame belonging to the time domain signal  $y$ . To sum up the signal synthesis, the PAPR minimization problem and the constraints for the two methods we get the QCQP in the following form:

minimize  $E$

subject to

$$[x \quad H_{F1} \quad H_{F2} \quad -I] \begin{bmatrix} 1 \\ t_{re} \\ t_{im} \\ a \end{bmatrix} = 0$$

$$E \geq |a|^2 \quad (10)$$

For FBMC case the same criterion on ACE and TR can be used as described in (8).

It can be shown that  $E$  is a convex function of  $t_{re}$  and  $t_{im}$ . Since  $H_{F1}t_{re} + H_{F2}t_{im}$  and the addition of  $x$  and subtraction of  $a$  are linear functions, they preserve convexity. Then the absolute value of the complex vector  $a = a_{re} + j \cdot a_{im}$  is squared, again preserving convexity. Thus  $E$  is a convex function of  $t_{re}$  and  $t_{im}$  which guarantees that the problem has a unique global optimum.

In the simulations three cases are covered: using only ACE or TR constraints and using both constraints at the same time which is denoted as TR-ACE.

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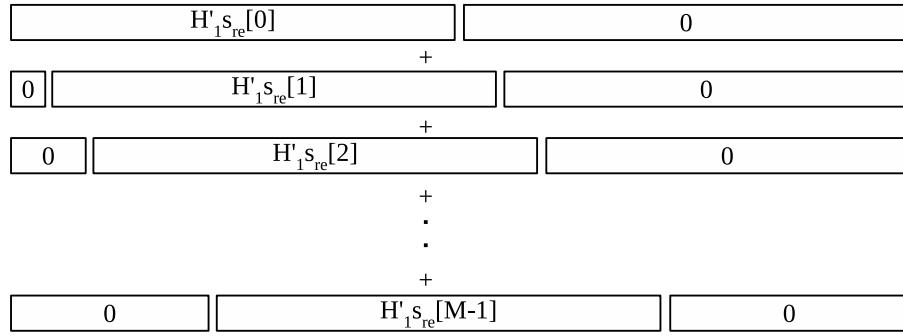


Fig. 4. FBMC frame synthesis using the real part of frequency domain symbols

## V. SIMULATION RESULTS

For the simulations we use the PHYDYAS prototype filters and the signal parameters described in Table I. In case of FBMC as the number of symbols per frame  $M$  increases, the complexity of the optimization problem described in (10) grows rapidly. Also in practical systems (e.g. 3GPP LTE) due to fine grained resource allocation frames consist of small number of symbols (in the order of tens). Thus for evaluation purposes we chose  $M = \{8, 12\}$  symbols per frame for FBMC signals. For a given frame size, the  $M$  symbols in the frame are jointly optimized which is needed because of the inherent overlapping of the signals. The results were obtained for both QPSK and 16QAM modulation alphabet. For the OFDM case the frame size is not relevant, since the problem can be solved on a per symbol basis. In order to achieve accurate PAPR values an oversampling rate of 4 was applied.

In each case the QCQP program was solved with MOSEK 7 [16] solver using the interior point method. For evaluation purposes we use the complementary cumulative distribution function (CCDF) of the PAPR which is calculated for OFDM

and FBMC based on (3) and (4), respectively. The results of PAPR reduction for OFDM with QPSK modulation is shown in Fig. 5. It can be seen that the TR and the ACE scheme alone at  $10^{-2}$  probability results in approximately 4.5 dB and 6 dB reduction, respectively whereas the TR-ACE brings less than 0.3 dB improvement, compared to using ACE only.

In Fig. 7 and Fig. 8 the PAPR reduction results for FBMC with QPSK modulation are shown for  $M = \{8, 12\}$  symbols per frame, respectively. The order of performance is the same as in the case of OFDM (in descending order: TR, ACE, TR-ACE), and the amount of improvement is approximately the same for both frame sizes which is 3 dB for TR, 6.2 dB for ACE and TR-ACE performs about by 0.3 dB better than using ACE only.

The results show that for QPSK modulation practically the same amount of PAPR reduction can be achieved in FBMC as in OFDM however, the lower limit for TR is approximately 1 dB higher in FBMC compared to OFDM. Thus the overlapping nature of FBMC is not a limiting factor when using TR or/and ACE schemes, and significant PAPR reduction can be achieved.

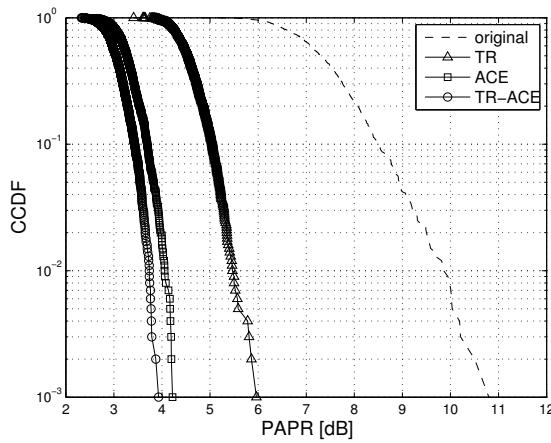


Fig. 5. CCDF of PAPR for original and PAPR reduced QPSK modulated OFDM signal

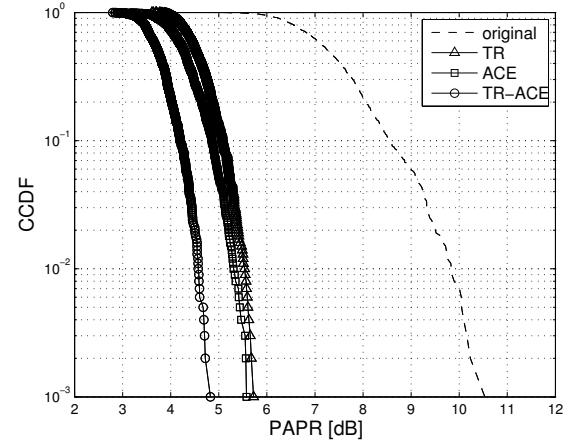


Fig. 6. CCDF of PAPR for original and PAPR reduced 16QAM modulated OFDM signal

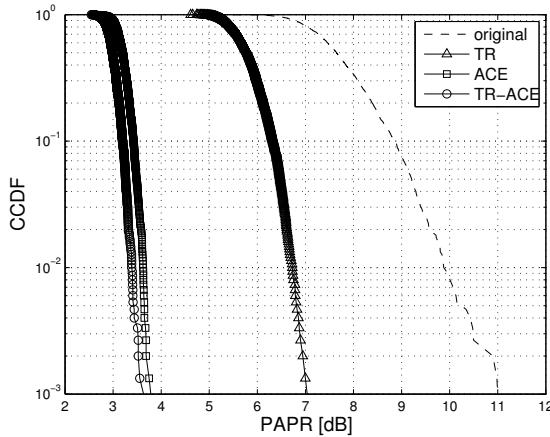


Fig. 7. CCDF of optimized PAPR with  $M = 8$  FBMC symbols per frame with QPSK modulation

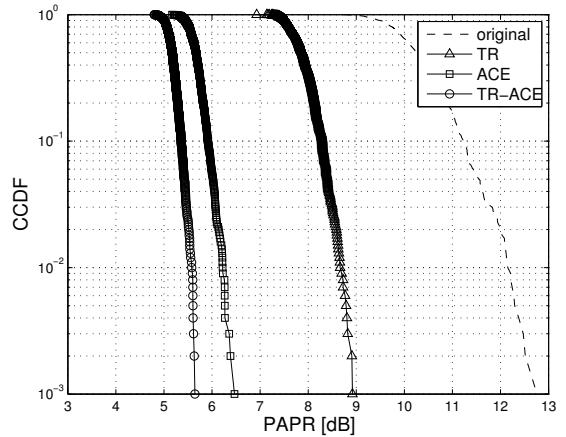


Fig. 9. CCDF of optimized PAPR with  $M = 8$  FBMC symbols per frame with 16QAM modulation

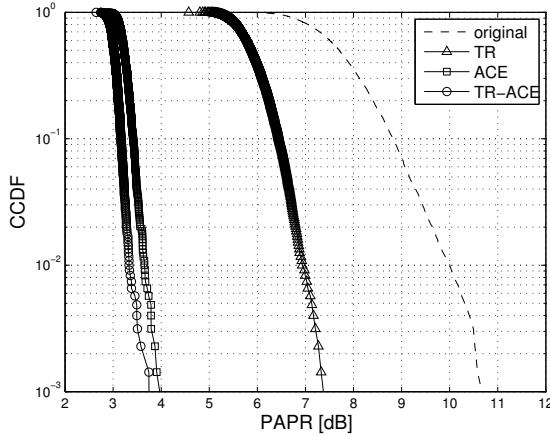


Fig. 8. CCDF of optimized PAPR with  $M = 12$  FBMC symbols per frame with QPSK modulation

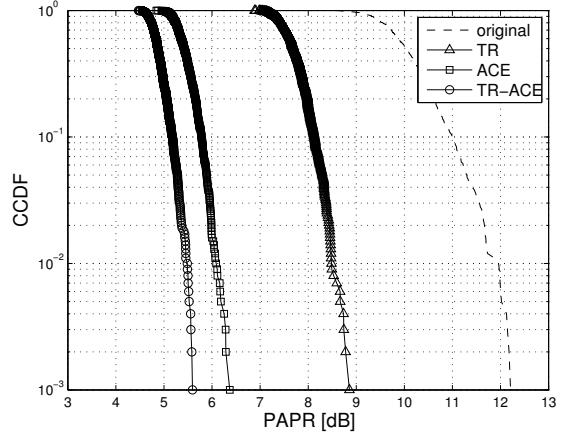


Fig. 10. CCDF of optimized PAPR with  $M = 12$  FBMC symbols per frame with 16QAM modulation

In Fig. 6 PAPR reduction results for OFDM with 16QAM modulation is shown. It can be seen that the TR method is more efficient for 16QAM than for QPSK, since there is 1 dB improvement in the reduction. Because the ACE scheme for this modulation alphabet has an effect only on the outer constellation points, thus it is less efficient than for the QPSK case (the difference is about 1.5 dB). The TR-ACE constraints are still the most effective and exhibit as much as 5 dB PAPR reduction (at  $10^{-2}$  probability).

For FBMC the results for 16QAM modulation for  $M = \{8, 12\}$  are shown in Figs. 9 and 10, respectively. In this case the results are very similar to the QPSK case, only the PAPR reduction capabilities are less by approximately 1.5-2 dB. This is also due to the ACE scheme has a limited set of constellation points to work on. For this modulation alphabet, there is no significant difference between the  $M = 8$  and  $M = 12$  cases.

Compared to the 16QAM OFDM the ACE and TR-ACE show impressive PAPR reduction potential, whereas the TR scheme is less efficient for FBMC, still delivering as much as 3 dB reduction.

## VI. CONCLUSIONS

In this paper we formulated the PAPR reduction problem for inherently overlapping FBMC signals. Based on this improvement we solved the QCQP for short FBMC frames and showed a lower limit of PAPR in presence of TR or/and ACE constraints. As a comparison we showed the results with the same parameters and constraints for OFDM signals. These results revealed that there is only minor difference in the performance of the PAPR reduction schemes between OFDM and FBMC. In case of QPSK approximately the same amount

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TABLE I. SIMULATION PARAMETERS

parameter	FBMC	OFDM
no. of subcarriers [N]	64	64
frame size [M]	8,12	1
overlapping factor [K]	4	0
no. of reserved tones [ $n_{res}$ ]	5	5
oversampling rate [V]	4	4
no. of realizations	1000	1000
modulation alphabet	QPSK, 16QAM	QPSK, 16QAM

of PAPR reduction can be achieved by using ACE or TR-ACE, however the TR method is at least 1 dB less efficient for FBMC. The lower bounds for 16QAM are higher in for both modulations, but in this case the ACE and TR-ACE schemes result in at least 0.5 dB less reduction compared to OFDM. Also the efficiency of TR for 16QAM is more than 3 dB less for FBMC. As the amount of reduction achieved by optimization is computationally complex, they should be considered as baselines for any real-time applicable algorithm developed using the aforementioned constraints.

## REFERENCES

- [1] P. Banelli, S. Buzzi, G. Colavolpe, A. Modenini, F. Rusek, and A. Ugolini, "Modulation formats and waveforms for 5G networks: Who will be the heir of OFDM?: An overview of alternative modulation schemes for improved spectral efficiency," *Signal Processing Magazine, IEEE*, vol. 31, pp. 80–93, Nov 2014.
- [2] S. Krishna Chaitanya Bulusu, H. Shaike, D. Roviras, and R. Zayani, "Reduction of PAPR for FBMC-OQAM systems using dispersive SLM technique," in *Wireless Communications Systems (ISWCS), 2014 11th International Symposium on*, pp. 568–572, Aug 2014.
- [3] N. van der Neut, B. Maharaj, F. de Lange, G. Gonzlez, F. Gregorio, and J. Cousseau, "PAPR reduction in FBMC using an ACE-based linear programming optimization," *EURASIP Journal on Advances in Signal Processing*, vol. 2014, no. 1, 2014.
- [4] Z. Kollar, L. Varga, B. Horvath, P. Bakki, and J. Bito, "Evaluation of clipping based iterative PAPR reduction techniques for FBMC systems," *The Scientific World Journal*, vol. 2014, p. 12, January 2014.
- [5] D. Qu, S. Lu, and T. Jiang, "Multi-block joint optimization for the peak-to-average power ratio reduction of FBMC-OQAM signals," *Signal Processing, IEEE Transactions on*, vol. 61, pp. 1605–1613, April 2013.
- [6] Y. Zhou, T. Jiang, C. Huang, and S. Cui, "Peak-to-average power ratio reduction for OFDM/OQAM signals via alternative-signal method," *Vehicular Technology, IEEE Transactions on*, vol. 63, pp. 494–499, Jan 2014.
- [7] B. Krongold and D. Jones, "PAR reduction in OFDM via active constellation extension," *IEEE Transactions on Broadcasting*, vol. 49, pp. 258–268, Sept. 2003.
- [8] J. Y. K. S.-E. Park, Y. Sung-Ryul, D. S. Park, and P. Y. Joo, "Tone reservation method for PAPR reduction scheme," tech. rep., IEEE 802.16e Task Group, IEEE 802.16e-03n60, Oct. 2003.
- [9] M. Petermann, D. Wubben, and K. D. Kammerer, "Joint constellation extension and tone reservation for PAPR reduction in adaptive OFDM systems," in *IEEE 10th Workshop on Signal Processing Advances in Wireless Communications, SPAWC '09*, pp. 439–443, 2009.
- [10] B. Horvath, Z. Kollar, and P. Horvath, "Bridging the gap between optimal and suboptimal ACE PAPR reduction scheme for OFDM," in *Radioelektronika (RADIOELEKTRONIKA), 2014 24th International Conference*, pp. 1–4, April 2014.
- [11] N. van der Neut, B. Maharaj, F. de Lange, G. Gonzalez, F. Gregorio, and J. Cousseau, "PAPR reduction in FBMC systems using a smart gradient-project active constellation extension method," in *Telecommunications (ICT), 2014 21st International Conference on*, pp. 134–139, May 2014.
- [12] S. Lu, D. Qu, and Y. He, "Sliding window tone reservation technique for the peak-to-average power ratio reduction of FBMC-OQAM signals," *Wireless Communications Letters, IEEE*, vol. 1, pp. 268–271, August 2012.
- [13] B. Farhang-Boroujeny, "OFDM versus filter bank multicarrier," *IEEE Signal Processing Magazine*, vol. 28, no. 3, pp. 92–112, 2011.
- [14] M. Bellanger, M. Renfors, T. Ihlainen, and C. A. F. da Rocha, "OFDM and FBMC transmission techniques: a compatible high performance proposal for broadband power line communications," in *Proc. IEEE Int Power Line Communications and Its Applications (ISPLC) Symp*, pp. 154–159, 2010.
- [15] Y. Dandach and P. Siohan, "FBMC/OQAM modulators with half complexity," in *2011 IEEE, Global Telecommunications Conference (GLOBECOM 2011)*, pp. 1–5, dec. 2011.
- [16] MOSEK ApS, *The MOSEK optimization toolbox for MATLAB*, 2015.



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