General Performance Analysis of Binary Fading Channels with Measurement Based Feedback Channel Equalization

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Abstract—**This paper presents a new general analytical technique for exact error rate analysis of coherent and noncoherent binary wireless communication systems in the presence of nonselective slow fading, additive white Gaussian noise (AWGN) and using measurement based feedback channel equalization technique by estimating the fading channel gain in an independent noisy pilot (measuring) channel. New exact bit error rate expressions are derived for coherent and orthogonal noncoherent binary system and the presence of different fading statistics. These results are derived in the case of signals experiencing Rayleigh, Rician or Nakagami-m fading. The results can be extended to arbitrary M-ary modulation.**

Index Terms—**Error analysis, binary wireless communication, fading mitigation with feedback, fading channel gain estimation, radio communication, closed loop power control.**

I. Introduction

The wireless and mobile technologies have improved significantly in the last two decades. In those years intensive investigations of wireless systems with closed-loop power control and channel equalization have carried out. This area in different contexts but always raised in every generation of mobile communication systems. The closed-loop power control solves the so-called near-far problem primarily, however the channel equalization compensates the effect of random fluctuation (fading) appearing in the radio channel. Traditionally, the prior problem is used to be solved with a high delay closed-loop power control system, while solution of the latter one requires diversity techniques ([5]). The closed-loop power control had an important role in the early mobile technologies (i.e. NMT and GSM), however its significance increased when the third generation CDMA systems were introduced. These systems require power control for the in-system interference coordination to provide reliable and fair communication. This topic is in focus since then, numerous papers were published ([6]-[20]) in the recent years. The closed-loop power control is also important in fourth generation systems (like LTE and its advanced versions) in connection with MIMO technologies ([21]-[29]). It has central role in the new direction of mobile communication systems, namely in the cognitive radio systems ([30]-[35]). This is motivated by the recent standardization trends towards permitting different heterogeneous communication systems to coexist and share a common wireless channel (e.g., unlicensed spectrum, opportunistic and dynamic spectrum access, spectrum underlay or overlay, ...).

To date, most research have focused on the performance of separated closed- loop power control system and diversity techniques to solve the near-far problem and the channel equalization in presence of white Gaussian noise and some special types of intentional interference. The most relevant to the present paper are those that analyses the error rate of the closed-loop power control scheme for the multi-rate services in the third and forth generation wideband systems. In [36] the authors demonstrate that the long scrambling pseudo-noise code, besides its well known feature in differentiating users and base stations, can improve power control false command over a frequency-selective fading channel as well. It is shown that the closed-loop power control error is a composite function of the spreading factor, target E_b/N_0 and Doppler frequency. In [37] the authors proposed an algorithm that computes the solution to the power control problem with closed-loop effects, and analytical and simulation results show that the algorithm converges under the same conditions as that given in the earlier results. In [38] the authors analyze the system performance of a truncated closed-loop power-control (TCPC) scheme for uplink in direct-sequence/code-division multiple-access cellular systems over frequency-selective fading channels. Closed-form formulas are successfully derived for performance measures, such as system capacity, average system transmission rate, MS average transmission rate, MS power consumption, and MS suspension delay. In [39] the authors propose smart step closed-loop power control (SSPC) algorithm in a DS-CDMA receiver in the presence of frequency-selective Rayleigh fading. This receiver consists of three stages. In the first stage the desired users' signal in an arbitrary path is passed and the inter-path interference (IPI) is reduced. Also in this stage, the multiple access interference (MAI) from other users is reduced. Thus, the matched filter (MF) can be used for the MAI and IPI reduction in the second stage. Also in the third stage, the output signals from the matched filters are combined according to the conventional maximal ratio combining (MRC) principle and then are fed into the decision circuit of the desired user. In [40] the performance of SIR (signal to interference ratio)-based closed loop power control (CLPC) is analytically analyzed. An analytical expression of the CLPC under fast fading is also produced. Finally a quantized-step size power control algorithm, replacing the hard limiter is considered.

The main aim of this paper is to present a unified analytical

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method for the accurate error rate analysis of coherent BPSK and orthogonal noncoherent BFSK in wireless communication systems with fast closed-loop power control systems subjected to general fading and using a measuring based channel estimation in a separated noisy channel. The main contributions are new exact expressions for error rates of coherent BPSK and orthogonal noncoherent BFSK in different fading environments (including Rayleigh, Nakagami-*m*, and Rician fading) and subjected to an non-exact channel transmission parameter measured in a noisy pilot channel. The results of this paper can be used to compare the performance of a coherent BPSK and orthogonal noncoherent BFSK wireless network that has a fast closed-loop power control system and uses diversity combining algorithms to parallel solve the near-far problem and channel equalization.

This paper is organized as follows. The basic model is described in Section II. Section III and Section IV present the unified method for the evaluation of the error probabilities in the case of coherent and noncoherent binary transmission. In Section V the average SNR is derived in the case of power limitation in the transmitter and in Section VI application examples are presented in the case of different fading parameters and power limitation factors. Section VII concludes the paper.

II. The Model

The lowpass equivalent complex-valued representation of the binary modulated signal $\mathbf{r}^*$ without channel equalization at the receiver front end during a signalling interval is given by

$$\mathbf{r}^* = \sqrt{E_s}\mathbf{g}^{(b)} z + \mathbf{n}, \quad t \in (0, T], \tag{1}$$

where E_s is the average symbol energy without fading and power control, T is the binary symbol time, $b \in [0,1]$ is the binary symbol and $\left\{\mathbf{g}^{(b)}, \ \left\|\mathbf{g}^{(b)}\right\|^2 = 1\right\}$ are the lowpass equivalent complex-valued representation of the elementary signals (in coherent case $\mathbf{g}^{(0)} = -\mathbf{g}^{(1)}$, and in noncoherent case $\left\langle \mathbf{g}^{(0)}, \mathbf{g}^{(1)} \right\rangle = 0$ typically), z is the complex fading channel gain and $\mathbf{n}$ is the lowpass equivalent complex-valued representation of the additive white Gaussian noise (AWGN) with $\mathbb{E}\left[\mathbf{n}\mathbf{n}^*\right] = N_0\mathbf{I}$, and $\mathbf{I}$ is the unity matrix. z is a RV that represents the instantaneous amplitude of the received binary signal ($\mathbb{E}\left[|z|^2\right] = 1$). The distribution of z depends on the fading scenario. In this paper, we consider three different types of nonselective slow fading scenarios, namely: Rayleigh, Rician and Nakagami-*m* fading models, and we suppose that the fading channel gain is estimated in the receiver by measurement in an independent pilot channel and the fading channel gain of the communication and measuring (pilot) channel are equivalent.

In our approach the fading channel gain is estimated at the receiver based on the use of an unmodulated pilot signal in an independent pilot (measuring) channel, therefore the lowpass equivalent complex-valued representation of the measuring signal at the receiver front end during a measuring interval is

$$\overline{\mathbf{r}} = \sqrt{E_0} z + \overline{\mathbf{n}}, \quad t \in \left(0, \overline{T}\right], \tag{2}$$

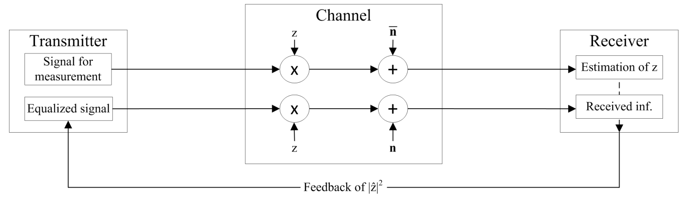

Figure 1. The model of our system. The fading channel gain of the measuring (pilot) and the communication channel are the same (z), but the noises ($\mathbf{n}, \overline{\mathbf{n}}$) are independent. The amplitude of the estimated fading channel gain ($|\widehat{z}|^2$) is fed back to the transmitter via an error-free digital communication channel. The transmitter equalizes the transmitted signal power based on this information.

where E_0 is the mean "symbol" energy of the measuring signal, $\overline{T}$ is the measuring time interval and $\overline{\mathbf{n}}$ is the lowpass equivalent complex-valued representation of the additive white Gaussian noise (AWGN) in the measuring channel with $\mathbb{E}\left[\overline{\mathbf{n}}\overline{\mathbf{n}}^*\right] = N_0\mathbf{I}$. In the paper we assume that the noises of the communication and the measuring (pilot) channel are independent ($\mathbb{E}\left[\mathbf{n}\overline{\mathbf{n}}^*\right] = \mathbf{0}$).

In our approach - for the fading suppression (channel equalization) - the transmitted signal is corrected by the estimated of the fading channel gain, where the best estimation of z is given by

$$\widehat{z} = \frac{\overline{\mathbf{r}}}{\sqrt{E_0}} = z + \frac{\overline{\mathbf{n}}}{\sqrt{E_0}}. \tag{3}$$

It is easy to show, that $\widehat{z}$ is a conditionally complex Gaussian RV if z is given, the mean value of $\widehat{z}$ is $\mathbb{E}\left[\widehat{z}\right] = z$ and the variance of it is given by

$$\mathbb{E}\left[\left(\widehat{z} - \mathbb{E}\left[\widehat{z}\right]\right)^2\right] = \frac{N_0}{E_0} = \frac{1}{\gamma_0}, \tag{4}$$

where γ_0 is the SNR of the measuring channel without fading. Note that we use an unmodulated pilot signal during the measurement process, hence the received energy via the pilot channel and therefore the accuracy of the fading channel gain estimation depends on the $\overline{T}$ too. If the measurement process and feedback delay is less than the fluctuation of the fading (what is assumed), the noise of the pilot channel ($\overline{\mathbf{n}}$) can be neglected.

In this paper we assume that the result of the measurement in the pilot channel ($|\widehat{z}|^2$) is fed back to the transmitter via a digital communication channel without any error and the transmitter uses this information to correct the transmitted signal power, as represented in Figure 1. The lowpass equivalent complex-valued representation of the corrected binary modulated signal at the receiver front end during a signalling interval is given by

$$\mathbf{r} = \sqrt{E_s}\mathbf{g}^{(b)} \frac{z}{\sqrt{h\left(|\widehat{z}|^2\right)}} + \mathbf{n}, \quad t \in (0, T], \tag{5}$$

where $\sqrt{h\left(|\widehat{z}|^2\right)}$ is the correction function at the transmitter, depending only on the cardinality of $\widehat{z}$, and $\mathbf{r}$ is a conditionally Gaussian RV, if z, $\mathbf{g}^{(b)}$ and $\overline{\mathbf{n}}$ are given. Using the elementary description of the communication systems the effective conditionally SNR of the communication channel is

$$\Gamma\left(\gamma_s|z,\overline{n}\right)=\frac{E_S}{N_0}\frac{|z|^2}{h\left(\left|z+\frac{\overline{\mathbf{n}}}{\sqrt{E_0}}\right|^2\right)}=$$

$$=\gamma_s\frac{|z|^2}{h\left(\left|z+\frac{\overline{\mathbf{n}}}{\sqrt{E_0}}\right|^2\right)}, \tag{6}$$

where γ_s is the SNR of the communication channel without fading, and throughout this paper we suppose that

$$h(x)=\begin{cases} c & \text{if } |x|\le c \\ |x|^2 & \text{if } |x|>c \end{cases} \tag{7}$$

is a simple threshold function considering an upper limit of the power in the transmitter.

Let us introduce the X and Y random variables as

$$X=|z|^2 \text{ and } Y=\left|z+\frac{\overline{\mathbf{n}}}{\sqrt{E_0}}\right|^2, \tag{8}$$

therefore

$$\Gamma\left(\gamma_s|z,\overline{n}\right)=\gamma_s\frac{X}{h\left(Y\right)}=\gamma_s\frac{X}{Y'}, \tag{9}$$

and

$$h\left(y\right)=y'=\begin{cases} c \text{ if } y\le c \\ y \text{ if } y>c \end{cases} \tag{10}$$

and determine the $f_{Y|X}\left(y|x\right)$ conditional probability density function of Y if X is given. After some simple mathematical manipulations one can get the following result:

$$f_{Y|X}\left(y|x\right)=\gamma_0\exp\left(-\gamma_0\left(x+y\right)\right)I_0\left(2\gamma_0\sqrt{xy}\right), \tag{11}$$

and

$$f_{XY}\left(x,y\right)=\gamma_0\exp\left(-\gamma_0\left(x+y\right)\right)I_0\left(2\gamma_0\sqrt{xy}\right)f_X\left(x\right) \tag{12}$$

therefore the pdf of Y is given by

$$f_Y\left(y\right)=\int_0^\infty\gamma_0\exp\left(-\gamma_0\left(x+y\right)\right)I_0\left(2\gamma_0\sqrt{xy}\right)f_X\left(x\right)dx, \tag{13}$$

where $f_X\left(x\right)$ is the pdf of the fading channel gain, and

$$f_X\left(x\right)=\exp\left(-x\right) \text{ in the case of Rayleigh,} \tag{14}$$

$$f_X\left(x\right)=\left(1+k\right)\exp\left(-k-\left(1+k\right)x\right) \tag{15}$$
$$I_0\left(2\sqrt{k\left(1+k\right)x}\right) \text{ in the case of Rician,}$$

$$f_X\left(x\right)=\tfrac{x^{m-1}}{x!}m^m\exp\left(-mx\right) \tag{16}$$
in the case of Nakagami-*m* fading models.

Using the definition of Y' (10) the pdf of Y' is given by

$$f_{XY'}\left(x,y'\right)=\begin{cases} A\left(x\right)\delta(y'-c)f_X\left(x\right) \text{ if } y'=c \\ f_{Y|X}\left(y'|x\right)f_X\left(x\right) \quad \text{if } y'>c \end{cases} \tag{17}$$

where

$$A\left(x\right)=\int_0^c f_{Y|X}\left(y|x\right)dy. \tag{18}$$

III. Derivation of the average error rate in the case of coherent binary transmission in fading channels

In this section, we present a new method for efficient computation of the average error probability of the coherent binary channels with the above mentioned feedback channel equalization by reducing the number of improper integrals. Our method is able to decrease the complexity of the mathematical problem and to increase the accuracy of the calculation. A key to the proposed method is to transform the conditional error probability function of the fading channel into a special form, which lets us use closed integral formulae and simplify the calculation of the average error rate. Therefore in the next subsection, we simplify the description of the proposed transformation technique, and prove its validity for different type of fading.

It is well known from the literature that the bit error rate of a binary coherent communication system is given by ([1])

$$P_b(\gamma_s|z,\overline{n})=\frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\mathrm{SNR}}\right), \tag{19}$$

where SNR is given in (9) and based on the well known lemma ([2]):

$$\frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\mathrm{SNR}}\right)=\frac{1}{\pi}\int_0^{\frac{\pi}{2}}\exp\left(-\frac{\mathrm{SNR}}{\cos^2\left(\theta\right)}\right)d\theta, \tag{20}$$

one can arrive at the following general expression of the conditional error probability of the coherent binary system:

$$P_b(\gamma_s|z,\overline{n})=\frac{1}{\pi}\int_0^{\frac{\pi}{2}}\exp\left(-\frac{\gamma_s\,x}{h(y)\cos^2\left(\theta\right)}\right)d\theta. \tag{21}$$

In this part of the paper, we present the new method for efficient computation of the average in (21). For that purpose let us use the Y and X RV-s (5), where Y is a conditionally Rician distributed random variable (if z is given) with pdf (11), as $\overline{\mathbf{n}}$ is a complex Gaussian RV with independent uniformly distributed real and imaginary parts.

Using (21) the conditional error probability function of the coherent receiver we can obtain the error function of P_b after calculating the average according to the Y and X random variables as follows:

$$P_b(\gamma_s)=\frac{1}{\pi}\int_0^{\frac{\pi}{2}}\mathbb{E}\left[\exp\left(-\frac{\gamma_s\,x}{h(y)\cos^2\left(\theta\right)}\right)\right]d\theta, \tag{22}$$

where

$$\mathbb{E}\left[\exp\left(-\tfrac{\gamma_s\,x}{h(y)\cos^2(\theta)}\right)\right]=$$
$$=\int_0^\infty\int_0^\infty\exp\left(-\tfrac{\gamma_s\,x}{h(y)\cos^2(\theta)}\right)f_{XY}\left(x,y\right)dxdy, \tag{23}$$

therefore the error probability can be calculated by the following triple integral (17):

$$P_b(\gamma_s)=\frac{1}{\pi}\int_0^\infty\int_0^\infty\int_0^{\frac{\pi}{2}}\exp\left(-\tfrac{\gamma_s\,x}{h(y)\cos^2(\theta)}\right)$$
$$f_{XY}\left(x,y\right)d\theta dxdy. \tag{24}$$

1) Rayleigh fading: In the case of Rayleigh fading model ($f_X\left(x\right)=\exp\left(-x\right)$) using (23) and (12) for the calculation of the error probability of the coherent system it is necessary to solve the following integral:

$$\int\limits_0^\infty \exp\left(-x\left(1+\gamma_0+\frac{\gamma_s}{h(y)\cos^2\left(\theta\right)}\right)\right)I_0\left(2\gamma_0\sqrt{xy}\right)dx, \tag{25}$$

and using the equation of [4, eq. (4.86)] we can get the following result:

$$\int\limits_0^\infty \exp(-at)I_0\left(2\sqrt{bt}\right)dt=\frac{1}{a}\exp\left(\frac{b}{a}\right), \tag{26}$$

where

$$a=1+\gamma_0+\frac{\gamma_s}{h(y)\cos^2\left(\theta\right)}\ \text{ and }\ b=\gamma_0^2y, \tag{27}$$

therefore one can arrive at the following final equation:

$$P_b(\gamma_S)=\frac{\gamma_0}{\pi}\int\limits_0^{\frac{\pi}{2}}\int\limits_0^\infty \frac{1}{1+\gamma_0+\frac{\gamma_s}{h(y)\cos^2(\theta)}} \exp\left(-\gamma_0 y\frac{1+\frac{\gamma_s}{h(y)\cos^2(\theta)}}{1+\gamma_0+\frac{\gamma_s}{h(y)\cos^2(\theta)}}\right)dyd\theta, \tag{28}$$

where the integral by y should be calculated in two intervals: $\{[0,c]\,;\ h(y)=c\}$ and $\{(c,\infty)\,;\ h(y)=y\}$.

2) Rician fading: In the case of Rice fading model using (23) and (12) for the calculation of the error probability of the coherent system it is necessary to solve the following integral ($f_X\left(x\right) = \left(1+k\right)\exp\left(-k-\left(1+k\right)x\right)I_0\left(2\sqrt{k\left(1+k\right)x}\right)$):

$$\int\limits_0^\infty \exp\left(-x\left(\left(1+k\right)+\gamma_0+\tfrac{\gamma_s}{h(y)\cos^2(\theta)}\right)\right) I_0\left(2\sqrt{k\left(1+k\right)x}\right)I_0\left(2\gamma_0\sqrt{xy}\right)dx. \tag{29}$$

Using the serial expansion of $I_0(.)$:

$$I_0\left(2\sqrt{k(1+k)x}\right)=\sum_{l=0}^\infty \frac{k^l\left(1+k\right)^l}{l!l!}x^l \tag{30}$$

this integral will be modified as follows:

$$\sum_{l=0}^\infty \tfrac{k^l(1+k)^l}{l!l!}\int\limits_0^\infty x^l I_0\left(2\gamma_0\sqrt{xy}\right) \exp\left(-x\left(\left(1+k\right)+\gamma_0+\tfrac{\gamma_s}{h(y)\cos^2(\theta)}\right)\right)dx, \tag{31}$$

and using the 6.643.2 and 9.220.2 equations of [3, eq. (4.86)] we can get the following final result:

$$P_b(\gamma_S)=\frac{\gamma_0}{\pi}\int\limits_0^{\frac{\pi}{2}}\sum_{l=0}^\infty\frac{k^l\left(1+k\right)^l}{l!} \int\limits_0^\infty {}_1F_1\left(l+1,1;\frac{\gamma_0^2\,y}{(1+k)+\gamma_0+\frac{\gamma_s}{h(y)\cos^2(\theta)}}\right) \frac{\exp\left(-\gamma_0 y\right)\exp\left(-k\right)}{\left((1+k)+\gamma_0+\frac{\gamma_s}{h(y)\cos^2(\theta)}\right)^{l+1}}dyd\theta, \tag{32}$$

where ${}_1F_1\left(.,.;.\right)$ is the confluent hypergeometric function.

3) Nakagami m-fading: In the case of Nakagami m-fading model ($f_X\left(x\right)=\frac{x^{m-1}}{\Gamma(m)}m^m\exp\left(-mx\right)$) using (23) and (12) for the calculation of the error probability of the coherent system it is necessary to solve the following integral:

$$\int\limits_0^\infty x^{m-1}\exp\left(-x\left(m+\gamma_0+\tfrac{\gamma_s}{h(y)\cos^2(\theta)}\right)\right) I_0\left(2\gamma_0\sqrt{xy}\right)dx, \tag{33}$$

and using the 6.643.2 and 9.220.2 equations of [3, eq. (4.86)] we can get the following final result:

$$P_b(\gamma_S)=\frac{\gamma_0 m^m}{\pi}\int\limits_0^{\frac{\pi}{2}}\int\limits_0^\infty\frac{\exp\left(-\gamma_0 y\right)}{\left(m+\gamma_0+\frac{\gamma_s}{h(y)\cos^2(\theta)}\right)^m} {}_1F_1\left(m,1;\frac{\gamma_0^2\,y}{m+\gamma_0+\frac{\gamma_s}{h(y)\cos^2(\theta)}}\right)dyd\theta, \tag{34}$$

where ${}_1F_1\left(.,.;.\right)$ is the confluent hypergeometric function.

To summarize, based on (24) the (28), (32) and (34) equations are new exact expressions for the average error probabilities of the coherent binary fading channels with measurement based feedback channel equalization over different fading channels.

IV. Derivation of the average error rate in the case of noncoherent binary transmission in fading channels

It is well known from the literature that the bit error rate of a noncoherent binary communication system with orthogonal elementary signals is given by ([1])

$$P_b(\gamma_s|z,\overline{n})=\frac{1}{2}\exp\left(-\frac{\text{SNR}}{2}\right), \tag{35}$$

where SNR is given in (9) and one can arrive at the following general expression of the conditional error probability of the coherent binary system:

$$P_b(\gamma_s|z,\overline{n})=\frac{1}{2}\exp\left(-\frac{\gamma_s\,x}{2h(y)}\right), \tag{36}$$

In this part of the paper, we present the noncoherent version of our new method for efficient computation of the average in (36). For that purpose let us use the Y and X RV-s (5), where

Y is a conditionally Rician distributed random variable (if z is given) with pdf (11), as $\overline{\mathbf{n}}$ is a complex Gaussian RV with independent uniformly distributed real and imaginary parts.

Using (36) the conditional error probability function of the coherent receiver we can obtain the error function of P_b after calculating the average according to the Y and X random variables, as follows:

$$P_b(\gamma_s) = \frac{1}{2}\mathbb{E}\left[\exp\left(-\frac{\gamma_s\, x}{2h(y)}\right)\right], \tag{37}$$

where

$$\mathbb{E}\left[\exp\left(-\frac{\gamma_s\, x}{2h(y)}\right)\right] = \int_0^\infty\int_0^\infty \exp\left(-\tfrac{\gamma_s\, x}{2h(y)}\right) f_{XY}\left(x,y\right)dxdy, \tag{38}$$

therefore the error probability can be calculated by the following double integral (17):

$$P_b(\gamma_s) = \frac{1}{2}\int_0^\infty\int_0^\infty \exp\left(-\frac{\gamma_s\, x}{2h(y)}\right) f_{XY}\left(x,y\right)dxdy. \tag{39}$$

1) Rayleigh Fading: Here, ξ is exponentially distributed, $f_\xi\left(x\right) = \exp\left(-x\right)$, and it can be shown using [3, eq. (6.614.1)] that

$$P_b(\gamma_S) = \frac{\gamma_0}{2}\int_0^\infty \frac{1}{1+\gamma_0+\frac{\gamma_s}{2h(y)}} \exp\left(-\gamma_0 y\frac{1+\frac{\gamma_s}{2h(y)}}{1+\gamma_0+\frac{\gamma_s}{2h(y)}}\right)dy. \tag{40}$$

2) Rician Fading: In this case ξ is distributed according to the non-central chi-square distribution, $f_\xi\left(x\right) = \left(1+k\right)e^{-k}\ e^{-(1+k)x} I_0\left(2\sqrt{k\left(1+k\right)x}\right)$ where k is the Rician factor, and $I_0\left(.\right)$ is the modified Bessel function of the first kind and zeroth order.

After similar calculations as in the case of coherent system one can get the following final result:

$$P_b(\gamma_S) = \frac{\gamma_0}{2}\sum_{l=0}^\infty \frac{k^l\left(1+k\right)^l}{l!}\int_0^\infty \frac{\exp\left(-\gamma_0 y\right)\exp\left(-k\right)}{\left(\left(1+k\right)+\gamma_0+\frac{\gamma_s}{2h(y)}\right)^{l+1}} {}_1F_1\left(l+1,1;\frac{\gamma_0^2\, y}{\left(1+k\right)+\gamma_0+\frac{\gamma_s}{2h(y)}}\right)dy. \tag{41}$$

3) Nakagami fading: In this case, ξ is a gamma RV with $f_\xi\left(x\right) = \frac{x^{m-1}}{\Gamma(m)}m^m e^{-mx}$, and it can be shown with the help of [3, eq. (6.631.1)] (equation 6.631.1.) that

$$P_b(\gamma_S) = \frac{\gamma_0 m^m}{2}\int_0^\infty \frac{\exp\left(-\gamma_0 y\right)}{\left(m+\gamma_0+\frac{\gamma_s}{2h(y)}\right)^m} {}_1F_1\left(m,1;\frac{\gamma_0^2\, y}{m+\gamma_0+\frac{\gamma_s}{2h(y)}}\right)dy, \tag{42}$$

where ${}_1F_1\left(.,.;.\right)$ is the confluent hypergeometric function, defined in [3, eq. (9.210)].

To summarize, based on (39) the (40), (41) and (42) equations are new exact expressions for the average error probabilities of the noncoherent binary fading channels with measurement based feedback channel equalization over different fading channels.

V. Derivation of the average SNR in the case power limitation

The closed-loop power control has an effect on the average transmission power. Namely, the average SNR decreases depending on the actual statistic of the fading channel gain, the estimation error of the channel parameter and the upper limit of the transmission power c. This subsection describes this effect in case of different fading types.

In the proposed system the instantaneous transmission power is calculated as follows:

$$E_t = \frac{E_s}{h(y)} = \frac{E_s}{h\left(\left|z+\frac{\overline{\mathbf{n}}}{\sqrt{E_0}}\right|^2\right)} = \frac{E_s}{y'}, \tag{43}$$

where E_s is the average symbol energy without fading and power control, y' is a realization of the Y' RV defined in (10). In this case the average SNR is given by the following expected value:

$$\mathbb{E}\left[\mathrm{SNR}\right] = \mathbb{E}\left[\frac{E_t}{N_0}\right] = \mathbb{E}\left[\frac{E_s}{N_0}\frac{1}{h(Y)}\right] = \gamma_s\mathbb{E}\left[\frac{1}{Y'}\right], \tag{44}$$

from which the average rise of the SNR can be calculated with the following expression:

$$\begin{aligned}\mathbb{E}\left[\frac{\mathrm{SNR}}{\gamma_s}\right] &= \mathbb{E}\left[\frac{1}{Y'}\right] = \int_0^\infty \frac{1}{y'}f_{Y'}\left(y'\right)dy' = \\ &= \int_0^c \frac{1}{c}f_Y\left(y\right)dy + \int_c^\infty \frac{1}{y}f_Y\left(y\right)dy = \\ &= \int_0^\infty\int_0^c \frac{1}{c}f_{XY}\left(x,y\right)dydx + \int_0^\infty\int_c^\infty \frac{1}{y}f_{XY}\left(x,y\right)dydx.\end{aligned} \tag{45}$$

1) Rayleigh fading: Here, ξ is exponentially distributed, $f_\xi\left(x\right) = \exp\left(-x\right)$, and it can be shown using [3, eq. (6.614.1)] that the average SNR is given by

$$\mathbb{E}\left[\frac{\mathrm{SNR}}{\gamma_s}\right] = \int_0^\infty \frac{1}{h\left(y\right)}\frac{\gamma_0}{1+\gamma_0}\exp\left(-\frac{\gamma_0}{1+\gamma_0}y\right)dy. \tag{46}$$

2) Rician Fading: In this case ξ is distributed according to the non-central chi-square distribution, $f_\xi\left(x\right) = \left(1+k\right)e^{-k}\ e^{-(1+k)x} I_0\left(2\sqrt{k\left(1+k\right)x}\right)$ where k is the Rician factor, and $I_0\left(.\right)$ is the modified Bessel function of the first kind and zeroth order, and after some mathematical manipulations ([3, eq. (6.631.1)], equation 6.631.1.) the average SNR can be calculated as

$$\mathbb{E}\left[\frac{\mathrm{SNR}}{\gamma_s}\right] = \int_0^\infty\sum_{l=0}^\infty \frac{k^l\left(1+k\right)^l}{l!}\frac{\gamma_0\exp\left(-\gamma_0 y\right)\exp\left(-k\right)}{h\left(y\right)} \frac{1}{\left(\left(1+k\right)+\gamma_0\right)^{l+1}}\,{}_1F_1\left(l+1,1;\frac{\gamma_0^2\, y}{\left(1+k\right)+\gamma_0}\right)dy, \tag{47}$$

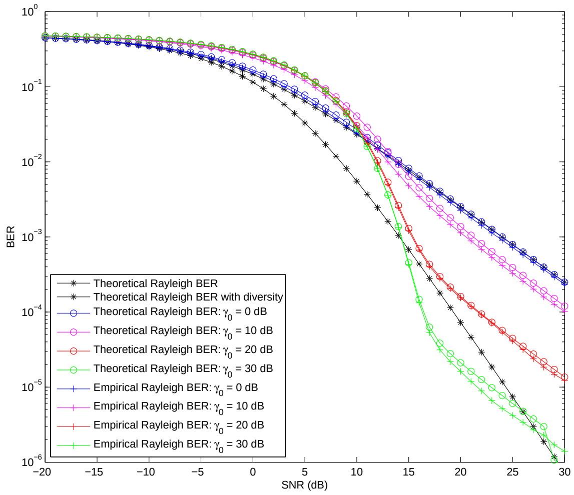

Figure 2. Validation of the theoretical results in the case of coherent binary transmission. Average BER of Rayleigh fading channel against the average SNR, when the SNR of the measuring channel is $\gamma_0 = 0 - 30$ dB and the power limiting factor $c = 0.001$.

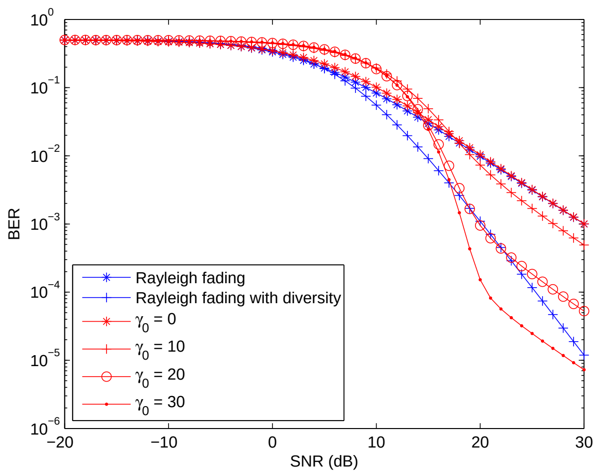

Figure 3. Theoretical results in the case of noncoherent binary transmission. Average BER of Rayleigh fading channel against the average SNR, when the SNR of the measuring channel are $\gamma_0 = 0 - 30$ dB and the power limiting factor is $c = 0.001$.

where ${}_1F_1\left(.,.;.\right)$ is the confluent hypergeometric function.

3) Nakagami Fading: In this case, ξ is a gamma RV with $f_\xi\left(x\right) = \frac{x^{m-1}}{\Gamma(m)} m^m e^{-mx}$, and it can be shown with the help of [3, eq. (6.631.1)] (equation 6.631.1.) that average SNR can be given by

$$\mathbb{E}\left[\frac{\mathrm{SNR}}{\gamma_s}\right] = \int_0^\infty \frac{1}{h\left(y\right)} \gamma_0 \frac{m^m}{\left(m+\gamma_0\right)^m} \exp\left(-\gamma_0 y\right) {}_1F_1\left(m, 1; \frac{\gamma_0^2\, y}{m+\gamma_0}\right) dy, \tag{48}$$

where ${}_1F_1\left(.,.;.\right)$ is the confluent hypergeometric function.

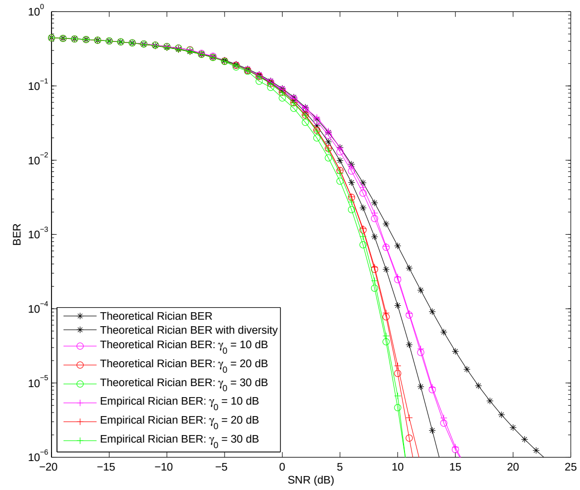

Figure 4. Validation of the theoretical results in the case of coherent binary transmission. Average BER of Rician fading channel against the average SNR, when the SNR of the measuring channel are $\gamma_0 = 0 - 30$ dB, $k = 10$ and the power limiting factor is $c = 0.001$.

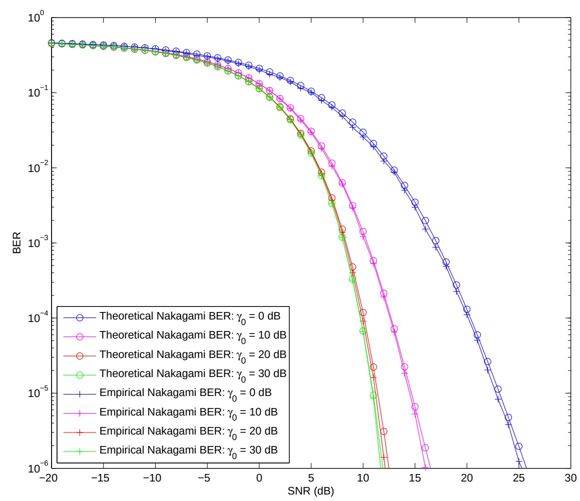

Figure 5. Validation of the theoretical results in the case of coherent binary transmission. Average BER of Nakagami fading channel against the average SNR, when the SNR of the measuring channel are $\gamma_0 = 0 - 30$ dB, $m = 4$ and the power limiting factor is $c = 0.001$.

VI. Numerical results and conclusions

This section of the paper introduces some numerical and simulation results and draws the most important conclusions. Fig. 2−Fig. 9 shows average Bit-Error-Rate probabilities against the average SNR of the communication channel in the interval of $-20 - 30$ dB. It is known from Section II, that the average BER depends on the fading channel gain z, the SNR of the measurement (pilot) channel γ_0 and the parameter of the transmission power limiting factor c. Therefore the results focus on the effect of these dependencies.

Fig. 2 and Fig. 3 show the average BER curves of Rayleigh fading channel against the average SNR in coherent and

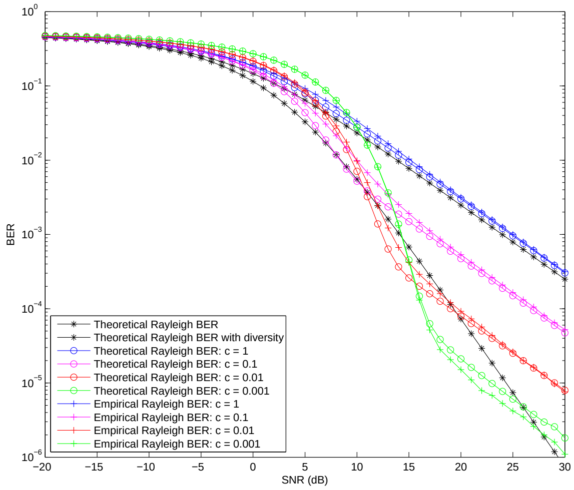

Figure 6. Validation of the theoretical results in the case of coherent binary transmission. Average BER of Rayleigh fading channel against the average SNR, when the SNR of the measuring channel is $\gamma_0 = 30$ dB and the power limiting factors are $c = 1 - 0.001$.

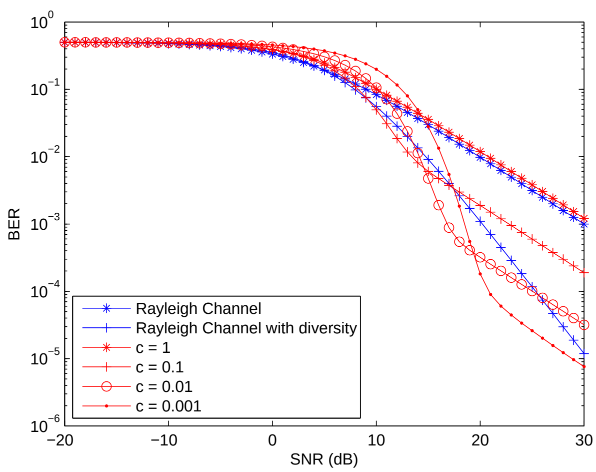

Figure 7. Theoretical results in the case of noncoherent binary transmission. Average BER of Rayleigh fading channel against the average SNR, when the SNR of the measuring channel is $\gamma_0 = 30$ dB and the power limiting factors are $c = 1 - 0.001$.

noncoherent case, respectively. The average BER curves of Rician fading channel in coherent case are shown Fig. 4, when the fading parameter is $k = 10$, while Fig. 5 represents the average BER curves of Nakagami fading channel in coherent case, when the fading parameter is $m = 4$, against the average SNR. In these figures the SNR of the measuring channel is $\gamma_0 = 0 - 30$ dB and the power limiting factor is $c = 0.001$. A straightforward consequence of the proposed system is, when the SNR of the measuring channel increases and therefore the uncertainty of the fading channel gain estimation decreases, that the average BER reduces.

Average probabilities of BER curves of Rayleigh fading channel against the average SNR in case of coherent and noncoherent case are represented in Fig. 6 and Fig. 7. Fig. 8

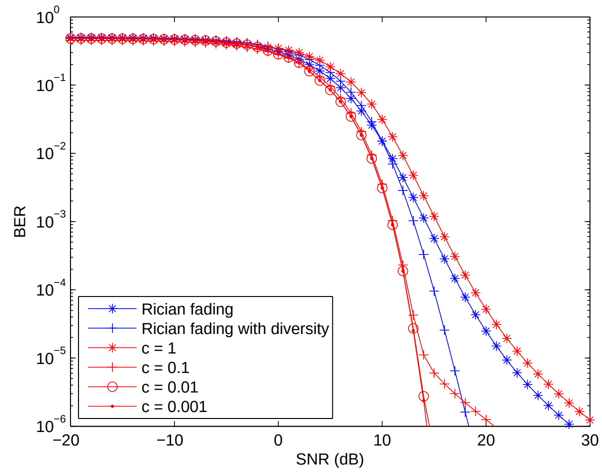

Figure 8. Theoretical results in the case of coherent binary transmission. Average BER of Rician fading channel against the average SNR, when the SNR of the measuring channel is $\gamma_0 = 30$ dB, $m = 4$ and the power limiting factors are $c = 1 - 0.001$.

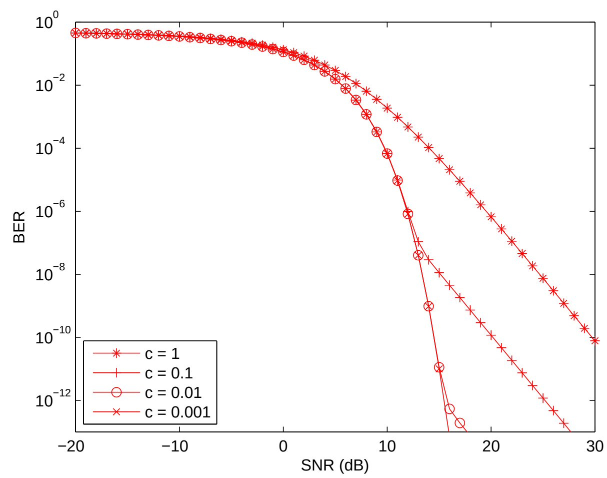

Figure 9. Theoretical results in the case of coherent binary transmission. Average BER of Nakagami fading channel against the average SNR, when the SNR of the measuring channel is $\gamma_0 = 30$ dB, $m = 4$ and the power limiting factors are $c = 1 - 0.001$.

shows the average BER curves of Rician fading channel in noncoherent case and Fig. 9 shows the average BER curves of Nakagami fading channel in coherent case against the average SNR. The fading parameters are the same as before ($k = 10$, $m = 4$). The SNR of the measuring channel is $\gamma_0 = 30$ dB and the power limiting factors are $c = 1 - 0.001$. Both in coherent and noncoherent case, the average BER reduces with the decrease of parameter c, which means with enabling higher transmission power the probability of bit error also decreases.

Note that the BER curve of the diversity is also represented. Because of using two channel (one for communication and one for the measuring) in our proposed system, it is better to compare the results against the diversity technique. The Fig. 2$-$Fig. 4 and Fig. 6$-$Fig. 8 show that it is possible to achieve better BER results instead of using diversity in some scenarios. That is true both coherent and noncoherent cases. This is expected when the fluctuation of the fading is slower than the measuring time interval ($\overline{T}$) and feedback delay because this property enables higher γ_0 values.

One can assume a basestation (BS) with given transmission power, which is able to communicate to the cell edge, and a simple path loss model. It is easy to understand that in this scenario one can transmit information with a low transmission limiting factor c in the close range of the BS. The parameter c is growing proportionately with the distance. However, based on the results it is better to use measurement based feedback channel equalization technique in close range instead of diversity.

The numerical results are validated by simulations. The simulation process is implemented in MATLAB, and in each scenario 10^8 transmission was evaluated.

As mentioned before, we assume an errorless and fast (faster then the fluctuation of the fading) feedback channel, which is a strict requirement. The effects of the delay and the error in the feedback channel are complicated problems, they needs further investigations.

VII. Summary

In this paper a new general analytical technique for exact error rate analysis of coherent and noncoherent binary wireless communication systems in the presence of nonselective slow fading, additive white Gaussian noise (AWGN) and using measurement based feedback channel equalization technique by estimating the fading channel gain in an independent noisy pilot (measuring) channel is proposed. It is shown with using measurement based feedback channel equalization, the proposed system is able to achieve BER gain against diversity technique in some scenarios, while the used channel bandwidth and the average transmission power is the same. The numerical results are validated by simulations.

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