Estimation of the Clutter Correlation Coefficient in Radar Systems

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Abstract. **Estimation of passive interference (so-called, a clutter), which is caused by the point objects, is considered for radar systems. The initial sequence of sampled correlated readings of the point (in distance) target is used as an initial one causing by the antenna beam scanning in a surveillance radar. Basing on the statistical description of this sequence, the likelihood function is introduced and its properties are discussed. An estimation algorithm of the clutter correlation coefficient is synthesized according to the sample of initial correlated readings using the maximal likelihood approach. A structural scheme is given for the correlation coefficient measuring system. The clutter correlation coefficient estimations obtained are asymptotically effective. On the base of the Cramer-Rao equation, the asymptotic formula for a variance of the correlation coefficient is derived, which determines a dependence of estimation accuracy on the value of the correlation coefficient and a number of averaged readings. The formula derived allows provision of necessary estimation accuracy by means of appropriate choice of the averaged reading number. Statistical modeling of the estimation algorithm is described and performed. Modeling results are given, which characterize a dependence of estimation accuracy of the clutter correlation coefficient upon the averaged readings number. A comparison of theoretical and empirical results for estimation accuracy analysis is performed. Statistical modeling results confirm the asymptotical character of the estimation accuracy of the clutter correlation coefficient.**

Keywords: correlation coefficient, clutter, estimation algorithm, estimation accuracy, statistical modeling

I. Introduction

Since the time of the Second World War and till the present days, passive interference is the effective measure of antiradar camouflage [1, 2]. Since then, one of the relevant and difficult problems of detection of moving target signals on the passive interference background remains invariably during design and operation of radar systems. Passive interference in the form of spurious reflections from the fixed or slowly moved objects (so-called, clutter): local objects, land or sea surface, hydrometeors (clouds, rain, hail, snow) and metallized reflectors dropped for target masking, essentially disturb the normal operation of various radar systems [2]. The clutter intensity may essentially exceed the level of the receiver inherent noise, which leads to overloading of the reception channel (so called, radar "blinding") and hence to useful signal missing. Nevertheless, even for overloading absence, the useful signal may be lost or not detected at all on the background of intensive spurious reflections.

Imperfection of analogous devices (ultrasonic delay lines, barrier-grid storage tubes) hindered a progress in development of protection means against the passive interference [3]. Application of digital signal processing allowed implementation of the sub-optimal processor on the base of a digital filter for clutter suppression with further discrete Fourier transform of samples [4-6]. Utilization of digital approaches leads to creation of rejection filters with adaptation to the clutter Doppler phase [7, 8]. Development of digital methods and devices for digital signal processing goes on at present to discuss in the modern scientific-technological literature [9-11].

Up-to date stage of this area development, *a priori* ambiguity of the spectral-correlation clutter characteristics as well as their heterogeneity and non-stationarity in the observation zone essentially hamper implementation of effective detection of moving objects on the clutter background, which stimulate innovations in radar systems and processing methods for radar signals. Overcoming of *a priori* ambiguity of clutter parameters is based on processing algorithm optimization depending on clutter parameters and further replacement of unknown parameters by their reasonable estimations [12] according to the adaptive Bayesian approach. These estimations can be obtained according to learning samples in the form of readings in the adjacent resolution bins in range od the Doppler frequency, which leads to adaptive algorithm and processing systems creation.

The adaptive detection of signals from movable targets on the clutter background, which is created by unwanted reflections from the lengthy objects, is described in [13]. The point (in range) interference corresponding (in range) to signals reflected from the point (in range) targets requires the special attention, which, in some cases, cannot be distinguished from the moving targets. Publications [14-17] are devoted to problems of surveillance radar system protection against this type of interference. That is why, it is interesting to have the estimation algorithms, which take into account the point character of the clutter.

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Estimates of maximal likelihood have found the most distribution since they are un-biased (or asymptotically unbiased) and asymptotically effective. In problems of signal detection, the clutter correlation coefficients [13] are, as a rule, the estimating parameters. At that, the detection effectiveness depends on estimation accuracy. In this connection, the estimation algorithm's choice and the point clutter accuracy analysis are a relevant issue. Algorithms and estimation accuracy of the clutter correlation coefficient are considered below according to the sample of the correlated readings of the point clutter.

II. SYNTHESIS OF THE ESTIMATION ALGORITHM

Now we consider a solution of stated problem in the coherent-pulse surveillance radar system, which performs the discrete of continuous space observation. Let it be a sequence of n sampled correlated readings of the point (in range) clutter at the output of linear portion of a receiver. This sequence is caused by the antenna beam scanning in the surveillance radar and presented in the following form: $\{u_j\}^{(n)} = \{u_1, u_2, \dots, u_n\}$. The estimation algorithm and the statistical estimation error result from statistical description of the given sample of readings caused by the fluctuation character of the spurious reflections.

Multiply reflection character leads to their Gaussian statistical description, being in this case adequate, the most reliable and experimentally confirmed [2]. The variation of reflection points, a height and range of the clutter source are taken into account in estimation results of required parameters, which is exactly the goal of the empirical approach to solution of this problem. Utilization of another model (not Gaussian) requires its reasonable choice experimentally substantiated and a presence of opportunities to describe the selective clutter properties, in particular, spectral properties. Use of the Markovian theory [18] for description of non-Gaussian signals turns out to be unpractical and unproductive. The Markovian model does not take into account the selective properties of signals and interference being as a matter of fact of non-coherent description and does not lead to practical circuits and devices, which allow extraction of moving target signals on the background of interference with surpassed power [18].

At Gaussian fluctuation character, the corresponding statistical description is given by a likelihood function (LF)

$$P(\{u_j\}^{(n)} / \rho) = (2\pi)^{-\frac{n}{2}} \det{}^{-\frac{1}{2}}[r_{jk}] \times \\ \times \exp\left(-\frac{1}{2} \sum_{j,\,k=1}^{n} w_{jk} u_j u_k \right), \qquad (1)$$

where $r_{jk} = \sigma^2 \rho_{jk}$ are elements of a clutter correlation matrix; σ^2 is the clutter variance; ρ_{jk} are correlation coefficients of the clutter readings; w_{jk} are elements of the reverse correlation matrix $[w_{jk}] = [r_{jk}]^{-1}$ of the clutter.

For interference in the form of the simply connected Markovian sequence, the correlation function has exponential character. Then, elements $\rho_{jk} = \rho^{|j-k|}$ and $r_{jk} = \sigma^2 \rho^{|j-k|}$. In this case

$$\det[r_{jk}] = \sigma^{2n} (1-\rho^2)^{n-1}, \qquad (2)$$

and elements of the reverse correlation matrix are

$$\left.\begin{aligned} & w_{11} = w_{nn} = 1/\sigma^2(1-\rho^2), \\ & w_{jj} = (1+\rho^2)/\sigma^2(1-\rho^2) \quad (1<j<n), \\ & w_{j-1,\,j} = w_{j,\,j-1} = -\rho/\sigma^2(1-\rho^2) \quad (1<j\le n). \end{aligned}\right\} \qquad (3)$$

Other elements w_{jk} equal to zero.

To solve this problem, we use the maximal likelihood method, which universality and relative simplicity combines with great achievements of estimations obtained, which are always (under condition of likelihood equation solution uniqueness) true, asymptotically normal and asymptotically effective [12, 19]. The estimation algorithm for the correlation coefficient ρ we find from the likelihood equation

$$\partial \ln P(\{u_j\}^{(n)}/\rho)/\partial\rho \Big|_{\rho=\hat\rho} = 0.$$

For this, at first, we take the logarithm from LF (1):

$$\ln P(\cdot) = -\frac{n}{2}\ln(2\pi) - \frac{1}{2}\ln\det[r_{jk}] - \frac{1}{2}\sum_{j,\,k=1}^{n} w_{jk} u_j u_k .$$

Taking into consideration the expression (2) and elements of the revere matrix (3), neglecting by edge effects of the main diagonal, we find

$$\ln P(\cdot) = -\frac{n}{2}\ln(2\pi) - \frac{1}{2}[\ln\sigma^{2n} + (n-1)\ln(1-\rho^2)] - \\ - \frac{1+\rho^2}{2\sigma^2(1-\rho^2)}\sum_{j=2}^{n} u_j^2 + \frac{\rho}{\sigma^2(1-\rho^2)}\sum_{j=2}^{n} u_{j-1}u_j . \qquad (4)$$

As a result of differentiation of (4), we have

$$\frac{\partial \ln P(\cdot)}{\partial\rho} = \frac{(n-1)\rho}{1-\rho^2} - \frac{2\rho}{\sigma^2(1-\rho^2)^2}\sum_{j=2}^{n} u_j^2 + \\ + \frac{2\rho^2}{\sigma^2(1-\rho^2)^2}\sum_{j=2}^{n} u_{j-1}u_j + \frac{1}{\sigma^2(1-\rho^2)}\sum_{j=2}^{n} u_{j-1}u_j . \qquad (5)$$

Now the likelihood equation has a form

$$\frac{(n-1)\rho}{1-\rho^2} - \frac{2\rho}{\sigma^2(1-\rho^2)^2}\sum_{j=2}^{n} u_j^2 + \\ + \frac{1+\rho^2}{\sigma^2(1-\rho^2)^2}\sum_{j=2}^{n} u_{j-1}u_j \Bigg|_{\rho=\hat\rho} = 0 .$$

After algebraic transformations, we obtain for the likelihood equation

$$(n-1)(1-\rho^2) - \frac{2}{\sigma^2}\sum_{j=2}^{n} u_j^2 + \\ + \frac{1+\rho^2}{\sigma^2\rho}\sum_{j=2}^{n} u_{j-1}u_j \Bigg|_{\rho=\hat\rho} = 0 .$$

For the greatly-correlated clutter ($\rho \to 1$), the likelihood equation takes a form

$$-\sum_{j=2}^{n} u_j^2 + \frac{1}{\rho}\sum_{j=2}^{n} u_{j-1}u_j \Bigg|_{\rho=\hat{\rho}} = 0\,,$$

from which we find the required estimation algorithm:

$$\hat{\rho} = \sum_{j=2}^{n} u_{j-1}u_j \Bigg/ \sum_{j=2}^{n} u_j^2\,. \qquad (6)$$

This algorithm results from the statistical synthesis procedure, in essence, is optimal and allows the simple physical interpretation. Accumulation presented in the algorithm allows smoothing of obtained sample fluctuations, thereby increasing the estimation accuracy. The algorithm (6) can be expanded to a case of the clutter with arbitrary correlation properties.

A structural scheme of the correlation coefficient measuring system is presented in Fig.1, which realizes the algorithm (6), where SD is the storage device, × is a multiplier unit, Acc is an accumulator, D is a divider unit.

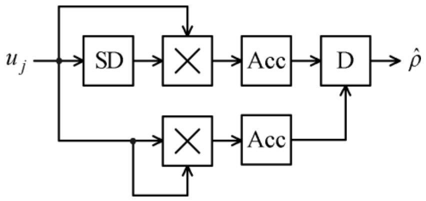

Fig. 1. A structural scheme of the correlation coefficient measuring system

III. ANALYSIS ON ESTIMATION ACCURACY

For future utilization of the correlation coefficient estimation $\hat{\rho}$, it is necessary to determine of its estimation accuracy. Taking into consideration that the obtained estimation $\hat{\rho}$ is asymptotically normal and asymptotically effective, we shall characterize the estimation accuracy by a variance of the correlation coefficient estimation $\hat{\rho}$, which is defined by the Cramer-Rao equation [19]:

$$\sigma_{\hat{\rho}}^2 = -\left[\overline{\partial^2 \ln P(\{u_j\}^{(n)} / \rho) / \partial\rho^2}\right]^{-1}. \qquad (7)$$

To make the appropriate calculations in (7), we find the second derivative of the likelihood function (1), by differentiation of (5):

$$\frac{\partial^2 \ln P(\cdot)}{\partial\rho^2} = \frac{(n-1)(1+\rho^2)}{(1-\rho^2)^2} +$$
$$+\frac{4\rho}{\sigma^2(1-\rho^2)^2}\sum_{j=2}^{n} u_{j-1}u_j -$$
$$-\frac{2(1+3\rho^2)}{\sigma^2(1-\rho^2)^3}\left(\sum_{j=2}^{n} u^2 - \rho\sum_{j=2}^{n} u_{j-1}u_j\right).$$

As a result of statistical averaging, taking into account that $\overline{u_{j-1}u_j} = \sigma^2\rho$, and $\overline{u_j^2} = \sigma^2$, we obtain

$$\overline{\frac{\partial^2 \ln P(\cdot)}{\partial\rho^2}} = \frac{(n-1)(1+\rho^2)}{(1-\rho^2)^2} + \frac{4(n-1)\rho^2}{(1-\rho^2)^2} -$$
$$-\frac{2(n-1)(1+3\rho^2)}{(1-\rho^2)^2} = -\frac{(n-1)(1+\rho^2)}{(1-\rho^2)^2}\,.$$

According to (7), we have finally:

$$\sigma_{\hat{\rho}}^2 = \frac{(1-\rho^2)^2}{(n-1)(1+\rho^2)}\,. \qquad (8)$$

As we see, the estimation accuracy depends on the correlation coefficient ρ and a number of averaged readings n. Evidently that the necessary accuracy of estimation can be provided by appropriate choice of the number of averaging readings n.

Expression (8) characterizes the potential accuracy of the measurement indicating the lower boundary of the variance $\sigma_{\hat{\rho}}^2$. Nevertheless, results of imitation statistical modeling of analyzed algorithms and signal processing devices are recognized by radar engineers as the most reliable and adequately represented the features of the actual devices.

IV. MODELING OF THE ESTIMATION ALGORITHM

Statistical modeling of the estimation algorithm includes formation of the model of an initial sequence of the clutter readings, calculation of the correlation coefficient estimation in accordance with the algorithm (6), and statistical determination of the estimate variance. Modeling is convenient to perform in the universal mathematical software package MathCAD, which is widely accepted as the best system for scientific-technological computations. The MathCAD package has powerful embedded means for implementation of numerical methods for calculations and mathematical modeling in combination with possibility to perform many operations of the symbolic mathematics.

Modeling of the initial clutter readings for the normal (Gaussian) distribution law and given correlation properties reduces to sequence formation of n sampled correlated readings.

To define a sequence of random numbers distributed by the normal law, we use the embedded element of random numbers, which is called in the MathCAD system by the function $\mathrm{rnorm}(m, \mu, \sigma)$, whose appropriate parameters are: m is a number of called elements, μ is mathematical expectation, σ is the rms deviation.

Formation of the initial sequence of correlated readings is done by means of linear transformation of random numbers, which are generated by the embedded element in the MathCAD system. In the case of the exponential correlation function $\rho_{jk} = \rho^{|j-k|}$, this linear transformation has a view:

$$u_j = \rho u_{j-1} + \sqrt{1-\rho^2}\,\mathrm{rnorm}(1,\ 0,\ 1)\,,\quad j = \overline{1,\ n}\,,$$

where $u_0 = \mathrm{rnorm}(1,\ 0,\ 1)$.

On the base of formed readings u_j, according to the algorithm (6), the correlation coefficient estimation $\hat{\rho}$ can be obtained.

By the method of statistical testing (the Monte-Carlo method) consisting of multiple repeat of the estimation algorithm (6) to estimate the correlation coefficient $\hat{\rho}$ we obtain the sample $\{\hat{\rho}_i\}$, $i = \overline{1, N}$, where N is a number of experiment repetitions.

The variance of correlation coefficient estimation is obtained from the following expression:

$$\sigma_{\hat{\rho}}^2 = \frac{1}{N-1}\sum_{i=1}^{N}(\hat{\rho}_i - \mu_{\hat{\rho}})^2 \cong \frac{1}{N}\sum_{i=1}^{N}\hat{\rho}_i^2 - \mu_{\hat{\rho}}^2,$$

where $\mu_{\hat{\rho}} = \frac{1}{N}\sum_{i=1}^{N}\hat{\rho}_i$ is the mathematical expectation or the mean value of the correlation coefficient estimation.

Figure 2 shows curves characterizing the dependence of the rms value of $\sigma_{\hat{\rho}}$ upon the readings number n at $\rho = 0.99$ and $N = 1000$. The solid curve corresponds to computations on the formula (8), and the dotted curve corresponds to empirical results obtained by means of statistical modeling on a computer.

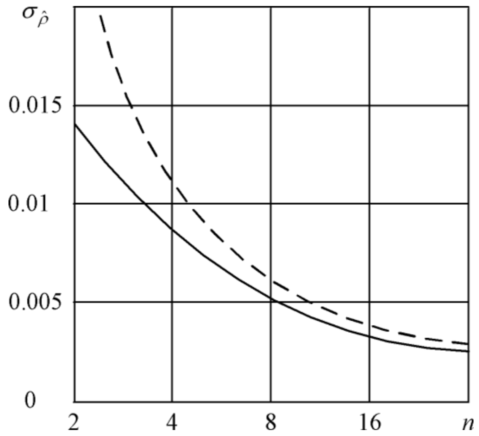

Fig. 2. Dependence of the rms value of $\sigma_{\hat{\rho}}$ upon the reading number n

Results obtained according to formula (8) have the asymptotical character approaching to the true measurement accuracy as n increase. Statistical modeling results confirm this circumstance. At $n = 8$ the modeling results differ from the calculation results basing on the formula (8) not more than by 20%. At $n > 8$ these differences are reduced accordingly, which confirms the asymptotical character

V. FUTURE INVESTIGATIONS

Results obtained in this paper are going to be used for optimization and analysis of the non-recursive rejection filterз.

In conformity with tasks of moving object selection on the background of lengthy passive interference, we are going to synthesize estimation algorithms for clutter correlation parameters: correlation coefficients and the Doppler phase shift, taking into account the interference structure. Moreover, we are going to analyze estimation accuracy depending on the clutter parameters and the volume of learning sample.

For rejection filters on non-recursive type, we hope to develop criteria and adaptation algorithms to unknown spectral-correlation characteristics of the clutter. On the base of approximating models of the clutter, we think to obtain stable (from computing point of view) algorithms of adaptive rejection of this clutter. We hope to develop structural schemes of adaptive rejection filters of sliding and grouped processing.

We are going to perform effectiveness analysis of non-recursive rejection filters versus the filter order, signal and clutter parameters, the volume of learning sample. We plan to solve problems of optimization under conditions of *a priori* ambiguity of signal detection systems on the background of the clutter, which perform the coherent rejection with further coherent or non-coherent accumulation of rejection remainders.

We hope to solve problems of parametric and structural optimization of detection systems for multi-frequency and non-equidistant signals on the clutter background. .

VI. CONCLUSION

The synthesized maximally-likelihood estimation algorithm (6) and the corresponding measuring device allow asymptotically effective estimations of the clutter correlation coefficient. The estimation accuracy, which characterize by formula (8) obtained for the clutter correlation coefficient, has the asymptotical character, which is confirmed by results of statistical modeling provided.

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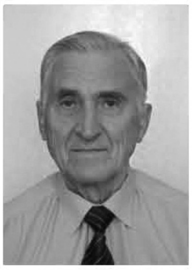

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